GEOMETRIC AND ANALYTIC ASPECTS OF GROUP THEORY

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It is well recognized that group theory, in its various forms, plays an important role in several areas of modern mathematics also extending into other sciences such as theoretical physics. Already Poincaré wrote in *Acta Mathematica* in 1915 that "la théorie des groupes est, pour ainsi dire, la Mathématique entière, dépouillée de sa matière et réduite à une forme pure."[1] Group theory is thus a natural meeting point for geometry, topology, analysis, dynamics, number theory, combinatorics, probability theory, etc. The present program reflects this diversity of connections, whether it be its geometric aspects reaching into Riemannian geometry, or its probabilistic features when groups serve as the stage for statistical mechanics models.

The subject of group theory has been a very active and prosperous one in recent decades, for example: Gromov's groundbreaking work in a variety of directions such as growth, decision problems, hyperbolic groups, etc. The use of property (T) to give first constructions of expander graphs and dynamics on Lie groups to solve long-standing open problems in number theory (by Margulis and others). The resolution of von Neumann's question on amenability and related questions (Grigorchuk, Olshanski-Sapir) and uniform growth (Wilson, Osin, Eskin-Mozes-Oh). The structure of linear groups e.g. the Breuillard-Gelander extensions of Tits alternative. Quasi-isometric rigidity (e.g. the recent work of Eskin-Fisher-Whyte). Andersen's proof that the mapping class groups do not have property (T). Bartels-Lück's proof of the Farrell-Jones conjectures for important classes of groups.

The present program focuses on exploring the geometric and analytic aspects of group theory. More specific topics are listed below, divided into two main strands:

1. Groups via their actions on topological and metric spaces:

I. Automorphism groups of free groups and Outer space; Mapping class groups and Teichmüller space.

[1] "group theory is, so to speak, all of mathematics, stripped of its content and reduced to a pure form" Acta Math, 38 (1915) p. 145.

II. Nonpositively curved spaces (CAT(0) spaces) and their isometry groups. Coxeter groups and generalizations. Lattices in Lie groups, symmetric spaces. Superrigidity.

III. Groups of automorphisms of rooted trees. Self-similar groups.

IV. Actions on the circle. Boundaries and boundary actions.

2. Groups as analytic-geometric-topological objects:

I. Homology and cohomology of groups. Bounded cohomology. Simplicial volume. Characteristic classes. Applications to affine structures, flat structures.

II. L^2-cohomology and ergodic theory. Spectral invariants. Random walks on groups. Percolation on Cayley graphs. Amenability. Kazhdan's property (T).

III. Asymptotic invariants of finitely generated groups. Algorithmic problems. Limit groups.

In conclusion, group theory, being the language of symmetry, is naturally ubiquitous in mathematics and other sciences. Recent decades have seen extraordinary developments in geometric and analytic aspects of infinite groups. A large number of difficult problems remain however to be solved and we hope that this program will make a significant contribution to further discoveries in the outlined directions.

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