Hamiltonians in Magnetic Fields

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Quantum magnetic systems play a fundamental role in contemporary science and technology. Many of the Nobel Prizes in Physics have been awarded for works on magnetic effects, starting with the 1902 prize to H. A. Lorentz and P. Zeeman for the discovery of the Zeeman effect, with the most recent one being the 2007 prize to A. Fert and P. Grünberg for the discovery of giant magneto-resistance.

The program takes as its unifying theme Hamiltonians in magnetic fields. Many problems involving only electric fields have received a satisfactory analysis, but the inclusion of magnetic fields poses new challenges, which often require new methods and techniques for their solution.

There are many different models, where the inclusion of a magnetic field is relevant and natural. For a single particle one has the Schrödinger operator, the Pauli operator, and the Dirac operator. For few particles the many-body Schrödinger and Pauli operators are relevant models. Note that in the presence of magnetic fields the spin of the particle becomes very important, due to the coupling between the spin and the magnetic field.

In quantum field theory there are many different models being proposed and investigated. The ultimate challenge is the full relativistic quantum field theory of electromagnetism (QED, quantum electrodynamics). It is providing us with extremely precise values for the anomalous magnetic moment of the electron, and for the Lamb shift of the energy levels of hydrogen. The rigorous study of the full QED seems to be far away, making it both relevant and challenging to study various simplified models. These include the Nelson model, the polaron model, models with infrared and/or ultraviolet cut-off, etc.

The magnetic field is also central in superconductivity. Here one of the simplified models, in this case a macroscopic model based on thermodynamical considerations, is given by the Ginzburg-Landau functional.

One can summarize the above in the statement that the goal is to find effective Hamiltonians that capture essential features of the model, sometimes in a limiting regime (for example large magnetic field), while at the same time the effective Hamiltonians are accessible to rigorous mathematical analysis.

Topics

The organizing committee has selected some topics, on which the program will concentrate its efforts to meet the challenge of inclusion of magnetic fields. In some of the topics there are strong groups in Scandinavia, while others represent the initiation of research in new areas for Scandinavian mathematicians.

Transport in quantum systems subjected to external magnetic fields has been an active area of research for a number of years. One of the central challenges is the understanding of the quantum Hall effect, in particular the case, where the edges are modelled by perturbations of the magnetic field.

Another challenge is the study of Anderson (de)localization for random magnetic fields. Many problems in this area are completely open. Bringing together several of the experts on the results obtained on random potentials is hoped to lead to substantial progress in this area.

The study of ultracold quantum gases in rotating traps leads to Hamiltonians with vector potentials, which mathematically are magnetic Hamiltonians, but the physical problem does not involve magnetic fields. `Artificial' magnetic fields can also be generated in optical lattices by rotating laser beams. These are very interesting recent models that will be studied from the rigorous mathematical point of view.

For Schrödinger type operators the dispersive estimates ($L^1 \to L^\infty$ estimates, Strichartz estimates etc.) have been obtained only in a few cases for the evolution with magnetic fields included. Inclusion of magnetic fields is a challenging problem, whose resolution will have a major impact on the study of those effective Hamiltonians that lead to non-linear Schrödinger type equations.

For Pauli and Dirac operators a challenge is the understanding of the zero modes. Open problems include criteria for the existence of zero modes, and their classification in terms of the geometry of the magnetic field. The zero mode problem depends on dimension. In dimension two it involves hard problems in complex analysis, but the general approach is known. In contrast, for dimension three a general approach is not known, and new methods need to be developed. Results on zero modes in dimension three are important in quantum field theory.

For Schrödinger type operators with magnetic fields the spectral theory poses new challenges. The magnetic quantum effects strongly affect their spectral and scattering properties, and new phenomena appear. In the study of spectral characteristics the effective Hamiltonian is often given as a Berezin-Toeplitz operator, and a detailed analysis of this class of operators becomes the challenge. Bringing together a number of experts at the Mittag-Leffler Institute is expected to lead to substantial progress concerning the quantization procedure, thus expanding the classical pseudo-differential operators to the magnetic case.

Stability of matter is the result that the ground state energy of a large Coulomb system consisting of N fermions has the property that the energy per particle is bounded below. It has been a central topic in rigorous atomic physics for the last 50 years, and many properties have been established. When one includes the quantized electromagnetic fields with a moment cut-off, the basic stability result has been obtained. One important application of stability of matter is to conclude the existence of the thermodynamic limit. For quantized magnetic fields, the question of existence of the thermodynamic limit remains an important open problem.

Recently magnetic fields have been included in the analysis of inverse problems, and this promises to be a very important new area. Inverse problems for magnetic operators are considered with the goal of recovering not only metrics and electric fields, but also magnetic fields, from either boundary measurements or from spectral and scattering data. One of the challenges is that the magnetic potential in the equations is unique only up to gauge invariance.

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