

## Stochastic Partial Differential Equations (SPDE's)

1 September - 15 December 2007 at Institut Mittag-Leffler

The theory of SPDE's is currently a very active field, with new and exciting results in its classical areas, but also with several new ideas for extending its domain of validity being actively investigated. The program aims at gathering representatives for these different trends of the subject.

Stochastic partial differential equations (SPDE's) is a topic in the area of stochastic analysis, at the crossing of stochastic processes and partial differential equations, with a huge development in the last fifteen years. The main motivations for the study of these objects come from interacting particle systems, superprocesses (limits of branching particle systems), dynamics in random environments, nonlinear filtering, control and potential theory. There is also a strong connection between SPDE's and deterministic parabolic and elliptic partial differential equations in infinite dimensions (Kolmogorov equations) and with Dirichlet forms.

SPDE's are a key ingredient of mathematical modelling in fields like physics, chemistry, biology and engineering. Many problems, like wave propagation in random media, turbulence, dispersion of flows in porous media, evolution of biological populations, can be now better understood using SPDE's. More recently the range of applications has been extended to oceanography, image analysis and mathematical finance among others. In fact, deterministic large scale models are not aware of small scale perturbation effects. A way to incorporate these fluctuations is by means of suitable random noises to the large scale dynamics.

One of the purposes of mathematical modelling is to predict and to get quantitative information on the phenomena. The statistical machinery provides a part of the tools. As far as SPDE's are concerned, this amounts to have a knowledge of the law of the solution. The stochastic calculus of variations, known as Malliavin calculus, can be used for that. Applications of Malliavin calculus to SPDE's are still at a preliminary stage; the work done until now refers, except for some particular examples, to elliptic linear differential operators. An indirect application of this calculus appears in SPDE's whose leading differential operator has random adapted coefficients. In fact, the mild form of the equation contains an anticipative stochastic integral. Recently the tools of Malliavin calculus have also been applied to study the qualitative behaviour of a class of SPDE's.

Malliavin calculus is one of the many examples of interplay between probability and mathematical analysis. In fact this interplay is giving fruits in both directions. Some new results on SPDE's have been proved or improved using probabilistic tools and on the other hand probability, and in particular SPDE's theory, has benefited from many tools and ideas coming from mathematical analysis. We plan along the semester to promote the communications between these two fields.

The choice of the right stochastic correction to the deterministic model is essential in the procedure of stochastic modelling. A future project is to apply the deterministic theory of rough paths analysis, developed by T. Lyons, to SPDE's driven by noises that could be very irregular, like fractional Brownian motion with any value of the Hurst parameter. Such models can be applied to analyse the microstructure of financial markets. In this context, an interesting issue is to develop numerical algorithms for the pricing of financial options with path dependent payoff, modeled through rough paths.

Very basic issues of the general theory of SPDE's, like existence and uniqueness of solution for nonlinear equations with non smooth coefficients are still not closed. The theory covers important examples, like the stochastic Navier-Stokes equation, the Langevin equation in Euclidean quantum field theory and SPDE's for the super-Brownian motion. The work by T. Souganidis in collaboration with P.L. Lions developing the basis of viscosity solutions for SPDE's could give more insight into these issues in connection with applications to control theory.

Most of the work done on SPDE's until now considers Gaussian noises. Recently, a new research line on SPDE's driven by Lévy processes is being developed. This is a challenging project, which entails new mathematical problems with a solid motivation in applied fields.

Finally, the semester plans to foster the connection between the general theoretical aspects of SPDE's and its applications, like the stochastic analysis of financial risk, stochastic climate modelling, stochastic models in biology. The composition of the steering committee reflects this aims, as well as the choice of invitees with expertise on these areas of applications and some others.

Scientific organizing committee:

Bernt Øksendal, University of Oslo

Michael Röckner, Universität Bielefeld

Marta Sanz-Solé, Universitat de Barcelona