STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

Workshop at the Mittag-Leffler Institute
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Abstracts of the talks

Vlad Bally, Université de Marne-la-Vallée, France

Lower bounds for the density of locally elliptic Itô processes

We consider a multidimensional Itô process dirven by an infinite dimensional Brownian motion. We assume regularity in Malliavin sense and we also make a local ellipticity assumption. Under these hypothesis the law of the process is absolutely continuous with respect to the Lebesgue measure and has a smooth density. Our aim is to give lower bounds for the densitiy which are of the same type as the ones obtained for elliptic diffusion processes.

References

[1] Bally, Vlad: Lower bounds for the density of locally elliptic Itô Processes. Annals of Probability, Vol. 34, No 6 - November 2006.

Nicolas Bouleau, ENPC, France

Bringing errors into focus

The lecture presents the main features of the theory of propagation of small errors as it may be thought today : The errors dichotomy. There are two kinds of small errors which do not follow the same differential calculus as shown by Monte Carlo simulations. Intrinsic error calculi. In finite dimension, the general (non necessarily symmetric) error calculi in $\mathbb{R}^n$ or in manifolds involve tangent vectors of order 2. Symmetric error calculus. Centeredness is not preserved by image. What is preserved is the symmetry of the error w.r. to the probability law. The error structures, using the language of Dirichlet forms, extend the error calculus to infinite dimension. The four bias operators yielded by an approximation and the Dirichlet form generated by the symmetric bias operator. Links with statistics. Fisher information and squared field operator. Provided that I succeed in improving current works, some of the above items will be replaced by insights into variational methods in optimization problems involving errors.

Zdzislaw Brzezniak, University of York, UK

On the stochastic Landau-Lifshitz' Equation (based on a joint work with B Goldys)

Motivated by a recent paper "Magnetic elements at finite temperature and large deviation theory" by R. V. Kohn, M. G. Reznikoff and E. Vanden-Eijnden, we are interested in stochastic parabolic Landau-Lifschitz equations. We investigate existence and uniqueness of solutions, as well as Large Deviations principle and stability.

Chris Burdzy, University of Washington, USA
Stochastic integration with respect to a quartic variation process (joint work with Jason Swanson)

The solution to the stochastic heat equation has the quartic scaling property in the time variable. I will discuss a research project leading to an Ito formula for processes with quartic scaling properties. The conjectured Ito formula is expected to contain the classical Ito integral as a "correction term" in this context.

Michael Caruana, University of Cambridge, UK
The existence of solutions for rough differential equations

In one of the last Saint Flour lectures in 2004, T. Lyons remarked that a Peano theorem for rough differential equations had not yet been proved. The first successful attempt to prove this existence result was carried out by A.M. Davie. Using generalized Euler schemes, Davie showed that when working in finite dimensions, differential equations driven by geometric p-rough paths, for $1 \leq p < 3$, have solutions whenever the vector fields have one degree less of smoothness than what is required for the existence of a unique solution in Lyons' Universal Limit theorem. P. Friz and N. Victoir have used geodesic approximations in the free nilpotent group to construct higher order Euler schemes and thus generalize Davie's results to the general case $p \geq 1$.
In this talk we present a new proof for Peano's theorem for rough differential equations, which is valid in infinite dimensions under an appropriate compactness assumption on the vector fields. Our approach, which is different from the ones mentioned above, makes full use of Lyons' Universal Limit theorem and is based on the construction of a family of rough polynomial approximations, each of which is a concatenation of rough path solutions of different equations.

Thomas Cass, University of Cambridge, UK
On the smoothness of density for solutions to stochastic diiferential equations with jumps

We consider solutions to SDEs including a diffusion component and a jump component which is introduced by an integral with respect to a Poisson random measure. We address the question of whether the law of such a solution possesses an infinitely differentiable density under Hoermander's conditions on the vector fields. We show that this is indeed true subject to some restriction on the rate at which the jump measure accumulates small jumps, but which nonetheless permits us to discuss a class of infinite activity jump processes. The central component of the proof is an extension of a classical semimartingale inequality of Norris.

Daniel Conus, EPFL, Switzerland

The non-linear stochastic wave equation in high dimensions

We present an extension of Walsh's classical martingale measure stochastic integral that makes it possible to integrate a general class of Schwartz distributions, which contains the fundamental solution of the wave equation, even in dimensions greater than 3. This leads to a square-integrable random-field solution to the non-linear stochastic wave equation in any dimension, in the case of a driving noise that is white in time and correlated in space. In the particular case of an affine multiplicative noise, we obtain estimates on p-th moments of the solution ($p \geq 1$), and we show that the solution is Hölder continuous. The Hölder exponent we obtain is optimal.
This is joint work with Robert C. Dalang, EPFL, Lausanne.

Hans Crauel, J. W. Goethe Universität, Germany

Measure attractors and Markov attractors

The actions induced by a random dynamical system on spaces of probability measures on the state space are investigated, and generalisations of the notion of an attractor are discussed and compared. For the particular case of a random dynamical system generated by a (ordinary or partial) stochastic differential equation the notion of an attractor for the associated Markov semigroup had previously been discussed in several instances in the literature. It is re-discovered here as a special case of a more general notion of an attractor in the space of measures.

Dan O. Crisan, Imperial College, UK

Particle approximations for linear parabolic SPDEs

The aim of the talk is to present some recent convergence results of a class of "hybrid" particle approximations to a class of linear parabolic SPDEs. These particle approximations combine the branching corrections approach introduced by Crisan and Lyons with the weighted approximation method of Kurtz and Xiong. The class of SPDEs plays the central role in nonlinear filtering: their solution is an unnormalized version of the conditional distribution of a partially observed Markov process.

Robert Dalang, Institut de Mathmatiques, EPFL, Switzerland

Hitting probabilities for systems of non-linear stochastic heat equations

We consider a system of $d \geq 1$ coupled non linear stochastic heat equations in spatial dimension $k \geq 1$, driven by d-dimensional spatially homogeneous Gaussian noise that is white in time. The solution of this system is a process indexed by space-time, with values in $\mathbb{R}^d$. The main objective is to determine, for a given subset of $\mathbb{R}^d$, whether or not this set is hit by the space-time process. This work in progress extends previous results obtained in [1, 2] in the case of spatial dimension 1 and space-time white noise. There, using Malliavin calculus, we obtained upper and lower bounds on the univariate and bivariate joint densities of the solution. These results lead to upper and lower bounds on hitting probabilities for the space-time process, in terms of capacity and Hausdorff measure of the sets. We also obtain related estimates when one of the space-time parameters is fixed. This makes it possible to determine the critical dimension above which points are polar, as well as the Hausdorff dimensions of the range of the process and of its level sets. Similar results were also obtained in [3] for systems of stochastic wave equations in spatial dimension 1.

References

[1] Dalang, R.C., Khoshnevisan, D., Nualart, E. Hitting probabilities for the non-linear stochastic heat equation with additive noise (preprint, 2006). http://arxiv.org/PS_cache/math/pdf/0702/0702710v1.pdf

[2] Dalang, R.C., Khoshnevisan, D., Nualart, E. Hitting probabilities for the non-linear stochastic heat equation with multiplicative noise (preprint, 2006). http://arxiv.org/PS_cache/arxiv/pdf/0704/0704.1312v1.pdf

[3] Dalang, R.C., Nualart, E. Potential theory for hyperbolic spde's, *Annals Probab.* **32** (2004), 2099–2148.

Laurent Denis, Université dEvry, France

Maximun principle for parabolic quasilinear spde

In this talk, we consider the following spde:

$$du_t(x) + Au_t(x) + f\left(t, x, u_t(x), \nabla u_t(x)\right) + \sum_{i=1}^{d} \partial_i g_i\left(t, x, u_t(x), \nabla u_t(x)\right)$$
$$= \sum_{j=1}^{d_1} h_j\left(t, x, u_t(x), \nabla u_t(x)\right)\, dB_t^j,$$

where A is a second order symmetric differential operator defined in some domain $\mathcal{O} \subset \mathbb{R}^d$, which satisfies ellipticity conditions and B is a B.M. We prove:

- L^p estimates ($p \geq 2$) for the uniform norm (in space and time) of the solution of this equation with Dirichlet condition on the boundary,
- that (local) solutions satisfy a maximum principle,
- comparison Theorems.

We consider the case where f, g and h are Lipschitz and then Burger's type equations (i.e. g may have polynomial growth).

References

[1] Denis L., Matoussi A. and Stoica, I. L. (2005) : L^p estimates for the uniform norm of solutions of quasilinear SPDE's, Proba. Theory and Related Fields 133, pp437-463.

[2] Denis L., Matoussi A. and Stoica, I. L. (2007) : Maximum principle and Comparison Theorem for Quasilinear Stochastic PDE's, preprint.

Mark Freidlin, University of Maryland, USA
Metastability and Stochastic Resonance in Multiscale Systems

General theory of metastability was developed in 1970-th in the framework of the large deviation theory. Stochastic resonance is a derivative of the notion of metastability. But if a system has more than one small parameter, and this is the case in many applications, there are many new effects and open questions. I will consider stochastic perturbations of systems close to Hamiltonian systems, systems with a small delay and some other multiparameter problems.

Peter Friz, University of Cambridge, UK
Malliavin calculus and rough paths

Solutions to rough path differential equations (RDEs) driven by Gaussian signals, including fBM with $H > 1/3$, are $\mathcal{H}$-differentiable. Under an ellipticity condition on the vector fields and a non-degeneracy condition on the Gaussian signal, solutions at fixed positive times admit a density.

Tadahisa Funaki, Tokyo University, Japan
SPDE with distributions of Levy processes as its invariant measures

We consider a heat equation on a half line with an additive noise chosen properly in such a manner that its invariant measures are a class of distributions of Lévy processes.

Martin Hairer, Warwick University, UK
Spectral gap results for a class of SPDEs

We give a survey of several techniques allowing to obtain spectral gap results for both finite and infinite-dimensional stochastic evolution equations. The talk will focus on highlighting different notions of "spectral gap" and the differences between the corresponding techniques. We will also provide an intuitive picture of the type of results that can be expected for various classes of problems, including some "negative" results about the lack of spectral gaps in some simple models.

O. Kutoviy, University of Bielefeld, Germany
Diffusion approximation for equilibrium Kawasaki dynamics in continuum

A Kawasaki dynamics in continuum is a dynamics of an infinite system of interacting particles in $\mathbb{R}^d$ which randomly hop over the space. In this paper, we deal with an equilibrium Kawasaki dynamics which has a Gibbs measure μ as invariant measure. We study a diffusive limit of such a dynamics, derived through a scaling of both the jump rate and time. Under weak assumptions on the potential of pair interaction, ϕ, (in particular, admitting a singularity of ϕ at zero), we prove that, on a set of smooth local functions, the generator of the scaled dynamics converges to the generator of an equilibrium diffusive dynamics of an infinite system of interacting particles. If the set on which the generators converge is a core for the diffusive generator, the latter result implies the weak convergence of finite-dimensional distributions of the corresponding equilibrium processes. In particular, if the potential ϕ is from $C^3(\mathbb{R}^d)$ and sufficiently quickly converges to zero at infinity, we conclude from a result in [Choi *et al.*, J. Math. Phys. 39 (1998) 6509–6536] that the convergence of process holds when the limiting diffusion is the gradient stochastic dynamics.

Xue-Mei Li, Loughborough University, UK
Why do we use Girsanov transform on no-linear filtering problem

Let p be a map between two space N and M. Given a Markov process (u) on N its projection to M is not necessarily Markov. The problem is to find the conditional law of u on its projection and the associated lift map. We allow u to be non-elliptic. The main tools are the symbols associated with diffusion operators, a decomposition of the diffusion operator associated to u and semi-martingale decompositions.

Arne Løkka, Kings College London, UK
Representation of positive distributions and characterisation of integrable random variables

One can show that a stochastic distribution (e.g. Hida distribution) is positive iff the S-transform of the distribution is positive definite. By appealing to Bochner-Minlos' theorem we can show that there exists a measure which completely characterise the chaos expansion of the stochastic distribution. Based on this representation, we proceed to show that there is an isometric isomorphism between the space of integrable functionals of Brownian motion and a certain space of signed finite measures. Finally I will briefly discuss some applications.

Sergey Lototsky, University of Southern California, USA
Stochastic parabolic equations of full second order

I will describe a procedure for defining a generalized solution of stochastic differential equations using the Cameron-Martin version of the Wiener Chaos expansion, and establish existence and uniqueness of this Wiener Chaos solution for stochastic parabolic equations with second-order differential operators in both the drift and the diffusion parts.

Utpal Manna, University of Wyoming, USA
Stochastic tidal dynamics equation

In this presentation I will mainly focus on the stochastic Tidal equation in 2-D. The proof of existence of strong solution using generalized Minty-Browder technique and exploiting global monotonicity property of the nonlinearity will be outlined. Here the use of global monotonicity avoids the classical method based on compactness hence the existence and the uniqueness results are new even in the deterministic case.

Bohdan Maslowski, Academy of Sciences, Czech Republic
Regularity of transition densities for parabolic SPDEs

Some earlier and recently obtained results on transition densities of Markov processes defined by semilinear parabolic SPDEs will be reviewed. In particular, regularity of densities and some regularization properties of transition semigroups will be discussed. Some applicataions of these results will be also mentioned.

Salah-Eldin A. Mohammed, Southern Illinois University USA
The Substitution Theorem for Semilinear Stochastic Partial Differential Equations

We gives an existence theorem and moment estimates for solutions of semilinear Stratonovich spde's with anticipating initial conditions. Our approach is to establish a substitution theorem for semilinear stochastic evolution equations (see's) depending on the initial condition as an infinite-dimensional parameter. Due to the infinite-dimensionality of the parameter and of the stochastic dynamics, existing finite-dimensional results do not apply. The substitution theorem is proved using Malliavin calculus techniques together with new estimates on the underlying stochastic semiflow. Applications of the theorem include dynamic characterizations of solutions of stochastic partial differential equations (spde's) with anticipating initial conditions and non-ergodic stationary solutions. The results are joint work with Tusheng Zhang.

Jan Pospisil, University of West Bohemia, Czech Republic
Ergodicity ane parameter estimates for infinite-dimensional fractional Ornstein-Uhlenbeck processes

Existence and ergodicity of a strictly stationary solution for linear stochastic evolution equations driven by cylindrical fractional Brownian motion are proved. Ergodic behaviour of non-stationary infinite-dimensional fractional Ornstein-Uhlenbeck processes is also studied. Based on these results, strong consistency of suitably defined families of parameter estimators is shown. The general results are applied to linear parabolic and hyperbolic equations perturbed by a fractional noise. Similar results apply also to linear evolution equations with a fractional noise in the boundary.

Michael Röckner, University of Bielefeld, Germany
Strong solutions of stochastic porous media equations: a survey of recent results

In the talk we shall give a survey on recent results for strong solutions of stochastic porous media equations, that is, stochastic partial differential equations of type

$$dX(t) = [\Delta\psi(t, X(t)) + \Phi(t, X(t))]dt + B(t, X(t))dW(t).$$

We shall cover the following topics: existence and uniqueness, asymptotic properties, regularity, preservation of positivity of initial conditions and Freidlin-Wentzell large deviations. We shall also compare our results with classical ones in the deterministic case.

Boris Rozovsky, Brown University, USA
On bi-linear elliptic SPDEs

Space-only noise is a natural random perturbation in equations without time evolution. Even the simplest equations driven by this noise often do not have a square-integrable solution and must be solved in special weighted spaces. The Cameron-Martin version of the Wiener chaos decomposition is an effective tool to study elliptic bi-linear SPDEs driven by space-only white noise. I will present some results about solvability of such equations in weighted Wiener chaos spaces (joint work with S. Lototsky).

Michael Scheutzow, Technical University Berlin, Germany
Attractors for stochastic flows on Euclidean spaces

We provide sufficient conditions for a random dynamical system (rds) on a Euclidean space to admit a random attractor. Roughly speaking, the drift vector field has to point towards the origin sufficiently strongly in order for an attractor to exist. We also provide an example of an rds generated by a stochastic differential equation, which does not possess an attractor even though its Markov semigroup admits a (unique) invariant probability measure. Finally, we relate attraction properties of a flow to expansion properties of the same flow run backwards in time. This is joint work with Georgi Dimitroff (Berlin).

Simone Scotti, Turin University, Italy
Application of error theory in finance

In classical theory of financial mathematics, we assume that all market securities have a definite price. Indeed, the hypothesis of completeness of the market forces a single price for a contingent claim. If we take into account an uncertainty on a parameter, we find that the price of the contingent claim is not unique but we have many possible prices.
We apply recent techniques, developped by Bouleau, to hedging procedures in order to perturbate parameters and stochastic processes, in the case of a volatility parameter fixed but uncertain for traders; we call this model Perturbed Black Scholes Model.
We show that this model can reproduce at the same time a smile effect and a bid-ask spread; we exhibit the volatility function associated to the local-volatility model equivalent to PBS model when vanilla options are concerned. Lastly, we present a connection between Error Theory using Dirichlet Forms and Utility Function Theory.

David Siska, School of Mathematics, Edinburgh, UK
On Randomized Stopping

We present a general result on the method of randomized stopping. We apply it to optimal stopping of controlled diffusion processes with unbounded coefficients to reduce it to optimal control problem without stopping. This is motivated by recent results of Krylov on numerical solutions to the Bellman equation.

Josef Teichmann, Technical University Vienna, Austria
Hypo-ellipticity in infinite dimensions

Some new extensions of results of Baudoin-Teichmann (AAP 2005) are provided and several examples are presented.

Johan Tysk, Uppsala University, Sweden
Feynman-Kac formulas for Black-Scholes type operators

There are many references showing that a classical solution to the Black-Scholes equation is a stochastic solution. However, it is the converse of this theorem that is most relevant in applications, and the converse is also more mathematically interesting. In this talk we establish such a converse. We find a Feynman-Kac-type theorem showing that the stochastic representation yields a classical solution to the corresponding Black-Scholes equation with appropriate boundary conditions under very general conditions on the coeffients. We also study the pricing equation in the presence of bubbles, ie when the price process is a strict local martingale. These results are obtained jointly with Svante Janson and Erik Ekström, respectively.

Jiang-Lun Wu, University of Wales Swansea, UK
On a Burgers type nonlinear stochastic equation

We consider a Burgers type nonlinear SPDE with Lévy space time white noise. Such an equation arises from white noise perturbation of nonlocal conservation laws. Existence and uniqueness of solutions will be discussed.

Tusheng Zhang, The University of Manchester, UK
Large deviations for stochastic non-linear beam equations

Large deviations for stochastic nonlinear beam equations. Abstract: In this talk, I will present a large deviation principle for the solutions of stochastic partial differential equations for nonlinear vibration of elastic panels (also called stochastic beam equations) which involve second order differentiation in time and cubic nonlinear terms.

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