APPLICATIONS OF STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS
Workshop at the Mittag-Leffler Institute November, 19-23, 2007

Abstracts of the talks

Robert Adler, Technion-Israel Institute of Technology, Israel
On quantifying shape, with two applications to stochastic processes

I shall discuss some classical Integral and Differential Geometric ways to classify shape, and describe

1. A new class of results about the excursion sets of smooth random fields which uses them.

2. An application of these classifiers to the study of the motion of manifolds under random flows.

Rainer Buckdahn, Université de Bretagne Occidentale. France
Stochastic viscosity solution for SPDEs driven by a Brownian motion

The talk is based on two recent common works with J.Ma/I.Bulla and J.Ma/N.Mrhardy, respectively, and will present a general approach for stochastic viscosity solution for nonlinear stochastic partial differential equations (SPDEs)

$$du(t,x) = F(x,(u,D_x u,D_x^2 u)(t,x))dt + g(x,(u,D_x u)(t,x)) \circ dW_t,$$

$t \in [0,T]$, $u(0,x) = u_0(x)$, $x \in R^d$, driven by a Brownian motion W; the stochastic integral is interpreted in the sense of Stratonovich. It generalizes former works by P.L.Lions and P.E.Souganidis as well as by J.Ma and R.B. investigating stochastic viscosity solution separately in two particular cases of the above equation: While in the case where g is independent of $D_x u$ a Doss-Sussman type transformation was applied in order to transform the SPDE into a pathwise PDE, the case of a u-independent diffusion term g was studied with the help of the method of stochastic characteristics. However, a generalization of former results by J.Ma and R.B. about the stochastic Taylor expansion allow to generalize the method of stochastic characteristics and to introduce the notion of stochastic viscosity solution for SPDEs in which the diffusion coefficient can depend on both u and its gradient $D_x u$. The talk will be completed by a uniqueness result for stochastic viscosity solutions.

Igor Chueshov, Kharkov University, Ukraine
Synchronization of a stochastic reaction-diffusion system on a thin two-layer domain
A system of semi-linear parabolic stochastic partial differential equations with additive space-time noise is considered on the union of thin bounded tubular domains joined at the common base. The equations are coupled by an interface condition which involves a reaction intensity $k(\epsilon)$, where ϵ is the thinness parameter of the domain. Limiting properties of the global random attractor are established as the thinness parameter of the domain goes to zero, i.e. as the initial domain becomes thinner, when the intensity function $k(\epsilon)$ is $o(\epsilon)$. In particular, the limiting dynamics is described by a single stochastic parabolic equation with the averaged diffusion coefficient, and nonlinearity term, which essentially indicates synchronization of the dynamics on both sides of the common base. Moreover, in the case of non-degenerate noise we obtain stronger synchronization phenomena in comparison with analogous results in the deterministic case.

Ana Bela Cruzeiro, Instituto Superior Tecnico, Lisbon, Portugal
Non ergodicity of the Euler flow on the torus
Brownian motions above the group of volume preserving diffeomorphisms of the d-dimensional torus are constructed. The asymptotic behaviour for large time of those processes shows the non existence of an invariant probability measure under the deterministic incompressible fluid dynamics. Nevertheless, in dimension 2, the energy induces on the group a Riemannian structure which has a positive renormalized Ricci tensor. (joint work with P. Malliavin, J. Funct. Anal. 2007)

Giulia di Nunno, University of Oslo, Norway
Lévy random fields: stochastic differentiation
We present some elements of stochastic calculus with respect to stochastic measures with independent values on a space-time product. In relation with the problem of finding explicit integrands in the Ito integral representation of square integrable random variables, we will consider stochastic differentiation both in the non-anticipating (Ito) and in the anticipating (Malliavin/Skorohod) framework. The relationship between the two will be exploited in order to obtain explicit formulae for the integrand. We will present and discuss in detail the derivative operators involved and their adjoint operators.

References
[1] Di Nunno, G.(2002) *Random Fields Evolution: non-anticipating integration and differentiation*, Theory of Probability and Math. Statistics, 66, 82-94.

[2] Di Nunno, G.(2007) *On orthogonal polynomials and the Malliavin derivative for Lévy stochastic measures*. Seminaires et Congres, 15.
[3] Di Nunno, G.(2007) *Random Fields: non-anticipating derivative and differentiation formulae*. Infin. Dimens. Anal. Quantum Probab. Relat. Top., 10, 465-481.

Boualem Djehiche, KTH, Stockholm, Sweden
A relaxed optimal control problem for nonlinear diffusions with application to bond portfolio choice.

We consider optimal control of a class of nonlinear diffusions of the McKean-Vlasov type where the controls which are measure-valued processes, enter both the drift and the diffusion coefficients together with the marginal law of the process. Existence results of an optimal control and necessary conditions for optimality, in form of a maximum principle, will be presented. The main motivation of this work comes from an optimal bond portfolio problem and trend identification problems in financial time series.
This is a joint work with Daniel Andersson.

Fred Espen Benth, University of Oslo, Norway
Spatial risk in the financial markets for temperature

We propose a continuous space-time model for the evolution of temperature, and apply this to problems if hedging and pricing in the financial temperature markets. The model calibrates data reasonably well, but yet simple enough to be analytically feasible be used for calculating prices for temperature forward contracts. Further, we consider the problem of hedging spatial temperature risk by use of synthetic temperature forwards.

Eric Gautier, ENSAE, France
Persistence of solitons for the stochastic Korteweg-de Vries equation

The stochastic Korteweg-de Vries equation is a model for the evolution of weakly nonlinear shallow water long waves. Solitons are particular solutions which are traveling waves and decay exponentially. They are stable against perturbation of the initial state. The stability of the soliton in the presence of noise, in KdV or the nonlinear Schrödinger equation, is an important issue. In [1], the validity of the approximation of the solution, when the initial datum gives rise to the soliton in the deterministic system, by a soliton with randomly fluctuating parameters is proved. This approximation is used very often in the physics literature. We will discuss the time scale, as function of the amplitude of the small noise, where this approximation holds. We will show that it is much larger than the exit time off a neighborhood of the deterministic solution. This phenomenon was previously studied numerically

by J. Printems and A. Debussche.
This is joint work with A. de Bouard.

References

[1] DE BOUARD, A., DEBUSSCHE, A. (2007): "Random modulation of soliton for the stochastic Korteweg-de Vries equation", *Ann. I.H. Poincaré-AN*, 24, 251-278.

Yaozhong Hu, University of Kansas, USA
Stochastic heat equation driven by fractional Brownian motion and intersection local time

In this talk we study a d dimensional sochastic heat equation with a multiplicative Gaussian noise which is white in space and has the covariance of a fractional Brownian motion with Hurst parameter $0 < H < 1$ in time. Two types of equations are considered. First we consider the equation in the Ito-Skorohod sense, and later in the Stratonovich sense. An explicit chaos development for the solution is obtained. On the other hand, the moments of the solution are expressed in terms of the exponential moments of some weighted intersection local time of the Brownian motion.

Peter Imkeller, Humboldt Universität Berlin, Germany
Martingale optimality, BSDE and cross hedging of insurance derivatives

A financial market model is considered on which agents (e.g. insurers) are subject to an exogenous financial risk, which they trade by issuing a risk bond. Typical risk sources are climate or weather. Buyers of the bond are able to invest in a market asset correlated with the exogenous risk. We investigate their utility maximization problem with respect to the correlation, and calculate bond prices using utility indifference. This hedging concept is interpreted by means of martingale optimality, and solved with BSDE tools. Prices are seen to decrease as a result of dynamic hedging. The increments are interpreted in terms of diversification pressure.

Torbjrn Kolsrud, KTH Stockholm, Sweden
Noether's theorem for linear and nonlinear diffusion

There is a variation principle for a large class of diffusions. This allows one to employ the Noether theorem, hence to produce conserved quantities. They may be identified with (local) martingales or conservation laws.

Stig Larsson, Chalmers University, Sweden
Finite element approximation of parabolic stochastic PDEs

We consider a linear parabolic stochastic differential equation in several spatial variables forced by additive colored space-time noise. The equations are discretized in the spatial variable by a finite element method. In order to capture the multiscale behavior of the colored noise we use a hierarchical wavelet basis for the finite element space. We prove strong convergence estimates.
This is joint work with Mihaly Kovacs.

Antoine Lejay, Université Henry Poincaré, Nancy, France
Young integrals and SPDEs

In this talk, we present a “functional” construction of Young integrals that allows one to solve in the sense of a mild solution some Stochastic Partial Differential Equations with an arbitrary noise, in a pathwise sense. Some constraints are imposed on the global regularity of the coefficients so that this approach cannot be used for white noise. Yet it can be used for some fractional noises.
This is joint work with Massimiliano Gubinelli and Samy Tindel.

Jorge León, Cinvestav-IPN, Mexico
Linear fractional differential equations

In this talk we compute the chaos decomposition of the solution to linear stochastic differential equations driven by a fractional Brownian motion with Hurst parameter less than 1/2. Here the coefficients are deterministic and the initial condition is a square-integrable random variable.

Christian Litterer, University College, Oxford, UK
Towards a high order method for non-linear filtering

We report on progress of our current research to apply the cubature on Wiener space method to the filtering problem.

Sergey Lototsky, University of Southern California, USA
Parameter estimation in diagonalizable bilinear stochastic parabolic equations

A parameter estimation problem is considered for a stochastic parabolic equation with multiplicative noise under the assumption that the equation can be reduced to an infinite system of uncoupled diffusion processes. From the point of view of classical statistics, this problem turns out to be singular not only for the original infinite-dimensional system but also for most finite-dimensional projections. This singularity can be exploited to improve the rate of convergence of traditional estimators as well as to construct a completely new closed-form exact estimator.

Salah Mohammed, Southern Illinois University, USA
A delayed option-pricing formula

In this talk we develop a formula for pricing European options when the underlying stock price follows a non-linear stochastic differential equation with memory. We expect the model for the stock dynamics to be sufficiently flexible to fit real market data, yet simple enough to allow for a closed-form representation of the option price (for long delays) as well as a hedging strategy. Furthermore, the model maintains the completeness of the market. The analysis is based on the construction of an equivalent martingale measure (no arbitrage) using a successive backward conditioning argument.
The results are joint work with M. Arriojas, Y. Hu and G. Pap.

Lluís Quer-Sardanyons, Universitat Autònoma de Barcelona, Spain
Existence and smoothness of the density for spatially homogeneous SPDEs

We extend Walsh's stochastic integral with respect to a Gaussian noise, white in time and with some homogeneous spatial correlation, in order to be able to integrate some random measure-valued processes. This extension turns out to be equivalent to Dalang's one. Then we study existence and regularity of the density of the probability law for the real-valued mild solution to a general second order stochastic partial differential equation driven by such a noise. For this, we apply the techniques of the Malliavin calculus. Our results apply to the case of the stochastic heat equation in any space dimension and the stochastic wave equation in space dimension $d = 1, 2, 3$. Moreover, for these particular examples, known results in the literature have been improved.

Carles Rovira, Universitat de Barcelona, Spain
On Itô's formula for elliptic diffusion processes

Bardina and Jolis proved an extension of Itô's formula for $F(X_t, t)$, where $F(x, t)$ has a locally square integrable derivative in x that satisfies a mild continuity condition in t, and X is a one-dimensional diffusion process such that the law of X_t has a density satisfying some properties. The formula was expressed using the quadratic covariation.
Following ideas of Eisenbaum for the Brownian motion, we show that one can re-express this formula using integration over space and time with respect to local times in place of quadratic covariation. We show also that when the function F has a locally integrable derivative in t we can avoid the mild continuity condition in t for the derivative of F in x.
This is joint work with Xavier Bardina.

Francesco Russo, Université Paris 13, Villetaneuse, France
Infinite dimensional calculus via regularization and one application to mathematical finance.

This talk generalizes some aspects of stochastic calculus via regularization introduced by F. Russo and P. Vallois to infinite dimensions. Moreover some applications to modeling financial assets without semimartinales will be performed. The stochastic integral intervening in the definition of self-financing property is forward integral.

Mikael Signahl, Oslo University, Norway

A stochastic NLSE with harmonic potential

We consider a defocusing nonlinear Schrödinger equation with a quadratic potential and perturbed by additive as well as multiplicative Wiener noise. Our aim is to show existence of an invariant measure on the statespace $H^1 \cap \mathcal{F}(H^1)$.

Josef Teichmann, TU Wien, Austria

Natural OU-processes on Lie groups with applications to simulated annealing

We show that a natural class of hypo-elliptic processes on Lie groups admits an invariant measure and a spectral gap with respect to it. We apply this class of processes to construct simulated annealing algorithms which converge in distribution to minima of non-convex functionals. The algorithms are non-elliptic and need therefore less independent Brownian motions than space dimensions. The universal constants depend on the geometry of certain nilpotent Lie groups. We apply the Driver-Melcher inequalities on Lie groups to show the main estimates.
Joint work with Fabrice Baudoin (Toulouse) and Martin Hairer (Warwick).

Roger Tribe, Warwick University, UK

2-parameter phase diagram for a stochastic reaction diffusion system

A stochastic reaction diffusion system describes the evolution of a branching population and its reaction with an underlying nutrient. Two parameters in the model control a phase diagram determining the large time behaviour of the population. This talk describes tools to study this phase diagram and some asymptotics for the critical curve for small and large parameter values.

Johan Tysk, Uppsala University, Sweden

The Black-Scholes equation and stochastic volatility

In many stochastic volatility models for option pricing the diffusion coefficients are allowed to grow faster than a quadratic polynomial. The corresponding Black-Scholes equations thus fall outside the standard classical theory. We consider existence and uniqueness results for such equations. The results are obtained together with Erik Ekstrm.

Esko Valkeila, Institute of Mathematics, Helsinki University of Technology, Finland

Approximation of geometric fractional Brownian motion

We give an approximation to geometric fractional Brownian motion in the sense of weak convergence, in the case when the self-similarity index of driving fractional Brownian motion is bigger than one half. Our approximation has the advantage to the previous ones in that the associated pricing model is free of arbitrage opportunities and complete.

Lorenzo Zambotti, Université Paris 6, France
Quasi-invariance properties of gamma processes

We present quasi-invariance formulae for gamma and Dirichlet processes under the action of non-linear path transformations. In the framework of von Renesse - Sturm, these formulae allow to define Dirichlet forms and associated reversible dynamics in the space of increasing functions from [0,1] onto itself. Such stochastic dynamics solve a SPDE with a complicated structure, which is still to be clarified.
This is joint work zith M.-K- von Renesse and M. Yor.

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