Complex analysis in several variables, Spring 2008 at the Mittag-Leffler Institute

Complex analysis in several variables has now been an active field in mathematics for more that one hundred years. During this time it has evolved into a rich subject in its own right, with tight links to many other branches of mathematics.

Ever since the beginning of the theory, the inhomogenuous $\bar{\partial}$-equation has played a main role - at first in the guise of an additive Cousin problem. The modern theory starts with the Kodaira vanishing theorem, followed by the work on the $\bar{\partial}$-Neumann problem of Morrey-Kohn and others and the weighted L^2-estimates of Hörmander and Andreotti-Vesentini. The weighted L^2-estimates still play a major role and continue to be extremely fruitful in applications, together with the use of two of their consequences - the Skoda division theorem and the Ohsawa-Takegoshi extension theorem - as exemplified by Siu's proof of the invariance of plurigenera. One feature of the recent developments here is the systematic use of singular metrics on holomorphic line bundles, and their associated multiplier ideal sheaves, introduced by Nadel. This has opened a very interesting connection between deep problems in algebraic geometry connected with the Mori program, and $\bar{\partial}$- and pluripotential theory.

Complex dynamics (iteration of holomorphic maps) have been studied since the 1920s, but the multidimensional theory has developed most rapidly in the last 20 years, to a large extent through it's connection with pluripotential theory and positive currents. This has e g lead to the construction of invariant measures of maximal entropy for classes of maps like Henon-maps and meromorphic maps in projective space. Many fundamental problems are nevertheless still open, as for example the existence of wandering Fatou components for Henon mappings.

Modern research on other geometrical aspects of several complex variables is essentially based on interactions with differential topology and especially with symplectic geometry. The fundamental idea in this respect was the introduction of pseudoholomorphic curves in Gromov's seminal paper. Existence of "enough" pseudoholomorphic curves, e.g. pseudo-holomorphic discs, gives obstructions in symplectic topology. New invariants emerge, as for instance, Floer homology. An important application is the proof of the Arnold conjecture on the number of fixed points of Hamiltonian symplectomorphisms in many cases.

Pseudoholomorphic curves have lead to spectacular results not only in symplectic geometry. These generalizations of holomorphic curves are more flexible than the latter and have already found applications in complex analysis. Examples are the study of envelopes of holomorphy using deformation of almost complex structure and the study of polynomial hulls. Another example of a fruitful interaction between complex analysis, symplectic geometry and real algebraic geometry is the recent proof of non-existence of Lagrangian embeddings of the Klein bottle into R^4 by complex analysts (Nemirovski, Shevshishin).

The spring semester at the Mittag-Leffler institute 2008 will primarily focus on four aspects of the theory:

The $\bar{\partial}$-equation and its applications in analytic and algebric geometry.
Complex dynamics in higher dimension.
Infinite dimensional complex analysis.
Complex geometry.

Bo Berndtsson, John-Erik Fornaess, Burglind Juhl-Joericke