Geometry, Analysis and General Relativity
Fall 2008 at Institut Mittag-Leffler

The Einstein equation is one of the most important geometric partial differential equations, in the Lorentzian as well as in the Riemannian setting. In the Lorentzian case, it is the field equation of general relativity and the mathematical study of it dates back to the time of its introduction by Einstein, who published the equations of general relativity in 1915. Among the mathematicians and physicists who have since then contributed in a fundamental way to our understanding of the Einstein equations, the names of Paul Dirac, Yvonne Choquet-Bruhat, André Lichnerowicz, Roger Penrose and Stephen Hawking deserve to be mentioned.

Mathematical research concerning Einstein's equation has made spectacular progress in recent years. This progress has taken place on many fronts, including the Cauchy problem, the cosmic censorship problem, construction of initial data, and the study of asymptotic behavior of the gravitational field. The area of contact between the Einstein equation and other geometric partial differential equations is large. Important classes of hyperbolic equations such as the wave maps equation and the Yang-Mills equation are connected with the Einstein equation at a fundamental level. Fluids as well as kinetic models such as the Bolzmann and Vlasov equations appear as matter models in general relativity.

The application of harmonic analysis techniques to the Yang-Mills equation, the wave maps equation, quasilinear wave equations and the Einstein equation has led to important breakthroughs concerning local well-posedness for rough initial data. These developments will be an important focus area during the programme. The proof by Christodoulou and Klainerman of the the nonlinear stability of Minkowski space was of fundamental importance, and rapid progress is now being made on a variety of nonlinear stability problems. Of these, the nonlinear stability of the Kerr black hole solution is of paramount importance, and poses a formidable challenge. As a step towards solving the Kerr stability problem, the asymptotic behavior of solutions of wave equations on black hole backgrounds is being studied by among others Dafermos and Rodnianski, and Finster, Smoller, Kamran and Yau.

There has been recent progress on the asymptotic behavior near singularities of solutions to nonlinear wave equations. Work by Struwe, Merle, Sigal, Bizon and others has yielded some of the first rigorous results in the hyperbolic case. The problem of analyzing the asymptotic behavior of solutions to the Einstein equations near singularities is closely related to the Cosmic Censorship problem. This problem has been solved in the Gowdy case. For more general situations, Cosmic Censorship is one of the most important open issues.

The Penrose inequality, which relates the mass of an initial data set to the area of horizons, was proved in the Riemannian case by Huisken and Ilmanen, using the inverse mean curvature flow, and by Bray using different techniques. The general case of the Penrose inequality remains open, and is just one of the fundamental problems relating to the notion of mass which will be studied during the programme. The inverse mean curvature flow has recently been used to provide new proofs of a number of geometric inequalities, including the isoperimetric inequality.

World leaders representing the different directions of research mentioned above will participate in the programme. Developments over the last few years have demonstrated deep connections among the topics mentioned above, and we expect that important new results will emerge from the interactions of the different specialties during the programme.

Program committee

- Lars Andersson, Albert Einstein Institute and University of Miami,
- Piotr Chruściel, University of Tours and Oxford University,
- Hans Ringström, Royal Institute of Technology,
- Richard Schoen, Stanford University