Discrete Probability, Spring 2009 at Institut Mittag-Leffler

Discrete probability is the study of stochastic models explicitly encompassing discrete consituents. Its history goes back all the way to a famous correspondence between Fermat and Pascal in 1654. Having been an active area of mathematical discourse ever since, its development has virtually exploded in the last few decades. This development is partly due the subject's own inner momentum, but perhaps even more to external influences from statistical mechanics, computer science, and other fields. The programme is intended to reflect these influences.

One of the main impacts that statistical mechanics has had on probability in recent decades is to shift some of the latter's focus away from proving classical limit theorems (laws of large numbers and central limit theorems) for dependent systems in greater and greater generality, and towards situations where the interactions between the system's constituents are so strong that the limit theorems actually break down in favor of other, more exotic, behaviors. A prime example is the Ising model - and more generally the Potts model - in two or more dimensions, that exhibit the two basic types of behavior (classical vs breakdown) depending on whether the temperature parameter is above or below a certain critical value. Such threshold phenomena (phase transitions) are nowadays paradigmatic to most of discrete probability, and much of the study focuses on what happens at or near criticality. Even the basic Ising model continues to be a rich source of exciting open problems.

The interface between computer science and discrete probability is currently a very hot area. A major example is the analysis of running times of algorithms - and not only randomized algorithms but also deterministic algorithms under probabilistic assumptions on their input data.

Besides the aforementioned Ising and Potts models, other benchmark models showing threshold phenomena are percolation and the Erdős-Renyi random graph, both of which are network models that have been studied for several decades and which exhibit remarkably rich behavior. More recently other network models have been proposed and analyzed, such as the Watts-Strogatz small-world network and its descendants, extending and partly a unification of percolation and the Erdős-Rnyi random graph. These new models, as well as others exhibiting a more "hierarchical" structure such as the preferential attachment model, have been heavily influenced by the ambition to find tractable mathematical models for real-world networks such as the Internet and the world-wide network of social contacts. In 2000, a seminal paper by Jon Kleinberg provided an algorithmic perspective on Watts-Strogatz-type models and revealed thitherto unknown difficulties in obtaining a satisfying mathematical explanation for the "six degrees of separation" phenomenon discovered in the 1960's by the experimental psychologist Stanley Milgram, and the hunt for more realistic models of real-world networks is still very much ongoing.

The subject of discrete probability is of course also intimately related to combinatorics, where "the probabilistic method" for obtaining existence results for various combinatorial objects continues to bear fruit.

The programme is intended to involve some of the world's best researchers in all of these facets of discrete probability. The rationale behind this decision is a combination of the observation that the subject as a whole has a sufficiently uniform language that we are all able to talk to and understand each other, and our conviction that chances for important progress are maximized by letting the various subfields interact with each other. In the same spirit, we intend the programme to encompass a wide variety of different techniques, ranging from purely probabilistic ideas such as coupling and Markov chain embeddings, to Fourier analysis and other analytic methods.

Let us finally emphasize that the heading "discrete probability" does not exclude the study of the various continuum objects that arise naturally as scaling limits of discrete objects. The most classical such object is Brownian motion which is obtained as the scaling limit of random walk. Another example, that was discovered by Oded Schramm in the late 1990's and that will most likely continue to have a huge impact, is the stochastic Löwner evolution (SLE) arising as a continuum limit of a wide variety of critical two-dimensional systems including percolation, Potts models and uniform spanning trees.

Program committee

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