

## Mathematical Logic: Set theory and model theory

We are living a golden age of set theory. Recent successes have led to surprising progress with classical questions like the Continuum Hypothesis. The success is now continuing with applications in topology, measure theory, algebra, and also in classical analysis.

The situation in set theory and the potential it offers is best manifested by highlighting the Inner Model Program. The *Axiom of Constructibility* of Kurt Gödel provides a rich axiomatic basis for set theory. However, it refutes large cardinal axioms. The *Inner Model Program* of set theory seeks generalizations which are compatible with large cardinal axioms. The program has been very successful and it has arguably yielded some of the deepest theorems of modern set theory to date. One notable success of the program has been the validation of determinacy axioms, particularly *projective determinacy*. This “solved” the classical questions of the projective sets raised in the early 1900s and it has yielded the axioms for second order number theory, axioms which are the true generalization of the Peano Axioms.

Despite these successes, in every case to date in which the program has succeeded, it has suffered from the same limitation; while the new axioms generalizing the Axiom of Constructibility are compatible with *some* large cardinal axioms, they are not compatible with large cardinals beyond the regime of large cardinal axioms which was originally specified. Moreover, the large cardinals beyond supercompact cardinals have seemed completely out of reach. Recently it has been shown that the current methodology *cannot work* for the large cardinals beyond the level of supercompact cardinals.

As a consequence of narrowing the focus of the Inner Model Program to this specific large cardinal axiom, one can state a series of conjectures which must be true if the Inner Model Program can succeed for this one large cardinal axiom. These conjectures concern infinitary combinatorics and have a priori nothing to do with the constructions of the Inner Model Program. This gives a completely independent probe of the possibilities for the success of the Inner Model Program at this level.

The development of the Inner Model Program to date has been closely linked to the developments in the other core areas of modern set theory, often in surprising and unexpected ways.

The ramifications of successfully advancing the Inner Model Program to the level of one supercompact cardinal, will be profound – for one will have discovered a generalization of the Axiom of Constructibility which is arguably compatible with essentially all large cardinal axioms. On the other hand if one can rule out success, then this also will have profound implications.

Recent results show that the progress made on generic absoluteness leads to completely new techniques in set theoretic model theory, especially in the study of model theory of strong extensions of first order logic. The theory of Abstract Elementary Classes is another recent development in set theoretic model theory. This is a promising new approach to studying the model theory of classes of structures that cannot be axiomatized in the traditional sense. Many examples from algebra and analysis belong to this category.

## Program Committee

- Erik Palmgren, Uppsala University
- Dag Normann, University of Oslo
- Jouko Väänänen, University of Amsterdam and University of Helsinki
- Hugh Woodin, University of California at Berkeley