Dynamics and Partial Differential Equations
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Dynamical systems originate with Poincaré and their study was pursued under different names during the first half of the 20th century by mathematicians like Lyapunov, Birkhoff, Siegel and the Russian school in qualitative theory of ordinary differential equations. This tradition culminated in the fifties and the sixties with the proof of the existence of invariant tori (Kolmogorov-Arnold-Moser, or "KAM"), the systematic study of hyperbolicity (Axiom-A) and structural stability and introduction of the horseshoe.

This early period is characterized by a domination of analytical and topological considerations. The theorem of Oseledec (from the late 60's) admits the existence of non-uniform hyperbolicity but its importance was not at all understood at the time. Even though the horseshoe establishes a relation between smooth dynamical systems and ergodic theory, the role of probability and ergodic theory remained quite limited. Another characteristic of the time is the almost exclusive concentration on low or finite-dimensional systems.

This changed in the middle 80's which saw important developments that set probability and ergodic theory at the center of smooth dynamics, and put focus also on infinite-dimensional systems. The program will focus on some of these developments.

Non-uniformly hyperbolic systems. The proof of the existence of the Hénon-attractor revealed the importance of non-uniformly hyperbolicity for dynamics. Since then the ergodic theory of Hénon-maps and Hénon-like maps have been intensively studied, and their relations to homoclinic bifurcations have been explored. Another non-uniformly hyperbolic system is the Kontsevitch-Zorich cocycle which is related to the Teichmüller flow and to interval exchange maps. The non-uniformly hyperbolic systems known today are either dissipative (like the Hénon-map) or co-cycles. The outstanding problem in this subject is to prove that the conservative so-called Standard map also has this property.

Quasi-periodic Schrödinger operators. In the 80's it was discovered that these operators, like random operators, admit pure point spectrum and localization. Since then, they have been intensely studied both by perturbative methods and by more global methods.

KAM-theory for PDE's. In the middle 80's KAM-theory was extended from Lagrangian tori to isotropic tori, then in the late 80's came the first proofs of quasi-periodic solutions for Hamiltonian PDE's. This has been a very active field since then, but many interesting problems are still open.

Theses themes have very strong connections to different versions of multiscale analysis, to renormalization and to ergodic theory (Oseledets theorem, Pesin's formula, the Ledrappier-Young theory). But there are also many other connections. One example is one-dimensional Schrödinger operators, which provide interesting examples of non-uniformly hyperbolic dynamical systems. Another example is the notion of Diophantine interval exchange maps (developed by Yoccoz et.al.) which relates these maps to KAM-theory.

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