On the structure and amoebas of discriminants

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It is known that the set of singularities of a nonconfluent hypergeometric function is solid. As a consequence, the zero set of any principle A–determinant has a solid amoeba. The same property is valid for the classical discriminant. In this talk we speak about new properties related to the classical discriminant, in particular, about the microlocal behavior of the Log–map and tentaclies of the amoeba.

We consider an univariate algebraic equation with variable complex coefficients. For the reduced discriminant locus ∇ of the equation there exist simple parametrizations for singular strata $M^{\nu} \subset \nabla$ corresponding to equations which have the only multiple root of the multiplicity ν. These parametrizations are the restrictions of the Horn–Kapranov parametrization of the whole discriminant set ∇ to a chain of nested linear subspaces of the projective space. We discuss the following statements.

- Strata M^{ν} can be transformed into the reduced A–discriminant set by monomial transformations, and, therefore, according to the result by J. Huh, the maximal likelyhood degree of M^{ν} is equal to one.

- The closure $\overline{M}^{\nu}$ touches $M^{\nu+1}$ in a cuspidal way. This property enables us to find on M^{ν} a simple rational expression for the multiple root of the initial equation.

Moreover, we prove that restrictions $\Delta|_{g_k}$ of the discriminant on noncoordinate hyperfaces g_k of the Newton polytope $\mathcal{N}(\Delta)$ are factorable into a product $\Delta_k \cdot \Delta_{n-k}$ of polynomial discriminants of degrees k and $n-k$.

The results were obtained in collaboration with I. Antipova and E. Mikhalkin.