ABSTRACTS

February Workshop at Institut Mittag-Leffler Programme "Spectral Methods in Mathematical Physics"

Horia Cornean
Title A Beals criterion for magnetic pseudo-differential operators proved with magnetic Gabor frames
Abstract First, we give a new proof for the Beals commutator criterion for non-magnetic Weyl pseudo-differential operators based on classical Gabor tight frames. Second, by introducing a modified magnetic Gabor tight frame, we naturally derive the magnetic analogue of the Beals criterion originally considered by Iftimie-Mantoiu-Purice. This is joint work with Bernard Helffer (Nantes) and Radu Purice (Bucharest),
https://arxiv.org/abs/1804.05220.

Michele Correggi
Title A Quantum Model of Feshbach Resonances
Abstract We consider a quantum model of two-channel scattering to describe the mechanism of a Feshbach resonance. We perform a rigorous analysis in order to count and localize the energy resonances in the perturbative regime, i.e., for small inter-channel coupling, and in the non-perturbative one. We provide an expansion of the effective scattering length near the resonances, via a detailed study of an effective Lippmann-Schwinger equation with energy-dependent potential.

Nils Dencker
Title The Solvability of Differential Equations
Abstract Since Hans Lewy sixty years ago presented his famous counterexample, it has been known that nonsymmetric linear partial differential equations are generically not solvable. For differential operators with simple characteristics, solvability is equivalent to the Nirenberg-Treves condition (Ψ). This condition involves the sign changes of the symbol of the nonsymmetric part of the highest order terms. When condition (Ψ) does not hold one also gets instability of the spectrum of the operator.

In this talk, we shall consider differential operators that have double characteristics. Examples are weakly hyperbolic operators and parabolic operators. Then one can define conditions corresponding to (Ψ) on the lower order terms at the double characteristics. We shall show that these conditions are necessary for solvability in several cases.

Maria Fialova
Title Qualitative analysis of two-dimensional Dirac operator with translationally invariant magnetic field
Abstract In the talk the two-dimensional Dirac operator with magnetic field of specific properties will be introduced as a selfadjoint operator. The notion of the constant fibre direct integral will be recalled. It turns out that with our requirements on the magnetic field the magnetic Dirac operator is decomposable. The emphasis is kept on the question of existence of the discrete spectrum of the so-called fibres of this operator in particular cases of parallel and perpendicular magnetisation. The existence of discrete eigenvalues of these fibres enables a mathematical construction of wave packets that are dispersionless in one direction.

Ira Herbst
Title Resonances in the one-dimensional Stark effect for small field
Abstract We consider the one-dimensional Schrödinger operator with compact support potential and an additional Stark field, fx. We locate and calculate the resonances at all locations. This is joint work with Richard Froese.

Mikael Hitrik
Title Positivity, complex FIOs, and Toeplitz operators
Abstract We establish a characterization of complex linear canonical transformations that are positive with respect to a pair of strictly plurisubharmonic quadratic weights. As an application, we show that the boundedness of a class of Toeplitz operators on the Bargmann space is implied by the boundedness of their Weyl symbols. This is joint work with Lewis Coburn and Johannes Sjöstrand.

Kenichi Ito
Title Commutator methods for the Stark Hamiltonian
Abstract We discuss spectral theory for a perturbed Stark Hamiltonian. The main results are Rellich's theorem, the limiting absorption principle, the radiation condition bounds and Sommerfeld's uniqueness result. For their proofs we adopt a commutator scheme from Ito-Skibsted (2016), in which choice of an escape function that conforms with classical mechanics of the Stark Hamiltonian plays a key role. This single escape function generates the conjugate operator and the Besov-type spaces. This talk is based on an ongoing joint work with T. Adachi, K. Itakura and E. Skibsted.

Simon Larson
Title Two-term spectral asymptotics for the Dirichlet Laplacian in a Lipschitz domain
Abstract In this talk I will discuss recent work on two-term spectral asymptotics for sums of eigenvalues of the Dirichlet Laplacian in a bounded open set with Lipschitz boundary. If time permits we shall also discuss how the methods developed can be used to obtain universal, i.e. non-asymptotic, bounds.

This is joint work with Rupert Frank.

Peter Madsen
Title Semi-classics of large fermionic systems in homogeneous magnetic fields.
Abstract We consider a system of N interacting fermions in the presence of a (strong) homogeneous magnetic field and with a semi-classical parameter $\hbar \sim N^{-1/3}$. The system is confined by an external potential, and the intensity of the interaction scales like $1/N$. I will describe the ground state energy of the system asymptotically to leading order using a magnetic Thomas-Fermi type functional. I will also discuss the weak convergence of approximate ground states of the system to the minimizers of the Thomas-Fermi functional. The analysis relies on the use of a classical de Finetti type theorem. The results extend those of a recent paper by Fournais, Lewin and Solovej.

This is joint work with Søren Fournais.

Marco Oliveiri
Title Derivation of time-dependent point interactions from polaron models
Abstract Time-dependent point interactions are model systems used to describe the phenomenon of ionization of a bound state by short- or zero-range potentials varying in time. The aim of the talk is to show how these models provide an effective approximation of the behavior of polaron microscopic systems in the regime of high intensity of the phonon field, which can be naturally reformulated as a suitable classical limit. Joint work with R. Carlone, M. Correggi and M. Falconi.

Georgi Raikov
Title Pauli operators with almost periodic electromagnetic fields
Abstract I will consider a two-dimensional Pauli operator H with an almost periodic electromagnetic field. First, I will discuss the general ergodic properties of H. Further, assuming that the electric potential vanishes, I will discuss the zero modes of H. A special attention will be allocated to the case of a vanishing mean value of the magnetic field. The talk is based on a joint work with J.-F. Bony (Bordeaux) and N. Espinoza (Tokyo).
The partial support of the Chilean Science Foundation Fondecyt under Grant 1170816 is gratefully acknowledged.

Nicolas Raymond
Title On the semiclassical spectrum of the Pauli operator with Dirichlet boundary conditions
Abstract This talk is devoted to the spectrum of the electro-magnetic Laplacian $(-ih\nabla - A)2 - hB$ on a bounded, regular, and simply-connected open set of the plane.

Here B is the magnetic field associated with A. We will see that, when the magnetic field is positive (and under generic conditions), we can accurately describe the low-lying eigenvalues in the semiclassical limit $h \to 0$. We will show the crucial role of the magnetic Cauchy-Riemann operators (and of their ellipticity), of the Bergman-Hardy spaces, and of the Riemann mapping theorem in the description of the spectrum.

This is a joint work with Jean-Marie Barbaroux, Loïc Le Treust, and Edgardo Stockmeyer.

Jonathan Rohleder
Hot spots of quantum graphs
Abstract The Hot Spots Conjecture of J. Rauch asserts that the hottest and coldest points of an insulated body should move towards its boundary for large times, if the insulation is perfect. Via the semigroup associated with the Neumann Laplacian this reduces to proving that maximum and minimum of the eigenfunction(s) associated with the smallest positive eigenvalue are located on the boundary. This conjecture is not true in full generality but is currently open, for example, for convex domains.

In this talk we will examine the corresponding question on metric graphs: for the Laplacian on a finite metric graph with standard (continuity and Kirchhoff) vertex conditions we consider the possible distribution of maxima and minima of eigenfunctions associated with the smallest nonzero eigenvalue. Among other things, we give examples to show that the usual notion of boundary of a metric graph, namely the set of vertices of degree one, has limited relevance for determining the hottest and coldest parts of a graph.

This is joint work with James Kennedy (Lisbon).

Grigori Rozenblum
Title Some spectral properties of the Neumann-Poincare operator in electrostatics and in elasticity (based on joint results with Y. Miyanishi, Osaka)
Abstract The Neumann-Poincare operator (callled often 'the double layer potential') was introduced in the end of XIX century and has been widely used since then. Recently, the interest towards this operator has revived,with relation to the plasmon resonance and cloaking in metamaterials (composite materials with some basic physical constants of the wrong sign). We discuss some recent results concerting the description of the essential spectrum and of the distribution of eigenvalues for this operator, for the electrostatic and elastic equations, with an interesting difference with a striking difference of results in dimension 2 and 3.

Erik Skibsted
Title Spectral and scattering theory of N-body systems at two-cluster thresholds.
Abstract We present and discuss results on various problems in spectral and scattering theory of N-body Schroedinger operators at a two-cluster threshold with particular emphasis on the usual (non-relativistic) models for atomic and molecular systems. This is part of an ongoing project with X.P. Wang.

Jeremy Sok
Dirac operators with magnetic links
Abstract The existence of zero modes for Dirac operators with magnetic fields is the cause of break down of stability of matter for charged systems. However the known examples are geometrically complex, and a complete classification of zero modes is unknown. In particular, one does not know the features of the magnetic fields which produce the zero modes.

We will present here some results corresponding to singular magnetic fields which are supported on links. These can be seen as generalizations of Aharonov-Bohm solenoids, in particular they exhibit the same 2π-periodicity of the fluxes carried by the field lines (the connected components of the given link). We have studied the occurrence of zero modes through the study of the spectral flow of loops of Dirac operators obtained by tuning one flux from 0 to 2π. We will show how the spectral flow depends on the geometry of the magnetic fields: the interlinking of the field lines but also their shape.

(Joint work with Fabian Portmann and Jan Philip Solovej)

Edgardo Stockmeyer
Title On the asymptotic dynamics of 2-D magnetic quantum systems
Abstract In this talk I will present results concerning the long time localisation in space (dynamical localisation) of certain two-dimensional magnetic quantum systems. The underlying Hamiltonian may have the form $H = H_0 + W$, where H_0 is rotationally symmetric and has dense point spectrum and W is a perturbation that breaks the symmetry.

(Joint work with Esteban Cárdenas, Dirk Hundertmark, and Semjon Wugalter.)

Tatiana Suslina
Title Homogenization of the stationary Maxwell system with periodic coefficients
Abstract We study homogenization of a stationary Maxwell system in $\mathbb{R}^3$ and in a bounded domain $\mathcal{O} \subset \mathbb{R}^3$ with sufficiently smooth boundary. The coefficients (electric permittivity and magnetic permeability) are periodic with respect to some lattice and depend on $\mathbf{x}/\varepsilon$. So, for small ε they oscillate rapidly. We are interested in the behavior of the solutions for small ε. The classical result is the weak L_2-convergence of the solutions to the solution of the effective problem, as $\varepsilon \to 0$. We find approximations for the solutions in the L_2-norm with error estimates of operator type.

Thomas Østergaard Sørensen
Title Properties of electron densities for Coulombic systems
Abstract We discuss various properties of the electron density associated to (exact) eigenfunctions for the Schroedinger operator of non-relativistic atoms and molecules. In particular, we discuss regularity - behaviour at and near the positions of the nuclei.

Timo Weidl
Trapped Modes in Elastic Media for Zero Poisson Coefficient

Dimitri Yafaev
Title Semiclassical asymptotic behavior of orthogonal polynomials
Abstract Our goal is to find asymptotic formulas for orthonormal polynomials $P_n(z)$ with the recurrence coefficients slowly stabilizing as $n \to \infty$. To that end, we develop spectral theory of Jacobi operators with long-range coefficients and study the corresponding second order difference equation. We suggest an Ansatz for its solutions playing the role of the semiclassical Green-Liouville Ansatz for solutions of the Schrödinger equation. The formulas obtained for $P_n(z)$ as $n \to \infty$ generalize the classical Bernstein-Szegö asymptotic formulas.

Kenji Yajima
Title Approximation of wave operators for point interactions by regular potentials.
Abstract We prove that the wave operators for Schrödinger operators with multi-center local point interactions are the scaling limits of the ones for Schrödinger operators with regular potentials. We simultaneously present a proof of the corresponding well known result for the resolvent which substantially simplifies the one in the book by Albeverio et al.