TITLES AND ABSTRACTS

March Workshop at

Institut Mittag-Leffler

Programme

”Spectral Methods in Mathematical Physics”

Patrick Gerard

Title: Inverse spectral theory for the Lax operator of the Benjamin-Ono equation

Abstract: The Benjamin-Ono equation is a nonlinear dispersive evolution PDE arising as an asymptotic model of long internal gravity waves in a stratified fluid. Its Lax pair structure was discovered in the late seventies. The Lax operator can be seen as a Toeplitz perturbation of the first derivative on the Hardy space.

In this talk, I will report on a recent jointwork with Thomas Kappeler about the (inverse) spectral theory of this Lax operator on the Hardy space of the disc, leading to the construction of global Birkhoff coordinates for the dynamics of the Benjamin-Ono equation with periodic boundary conditions.

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Rowan Killip

Title: Invariance of white noise under KdV

Abstract: I will outline a proof that the KdV flow on the line preserves white noise measure. The underlying problem here is to construct dynamics for such initial data, which has very low regularity and no decay at infinity. I will also discuss a new proof of the invariance of white noise under KdV on the circle, which was shown previously by Oh, Quastel, and Valko.

This is joint work with Jason Murphy and Monica Visan.

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Serena Cenatiempo

Title: Optimal Rate for Bose-Einstein Condensation in the Gross-Pitaevskii Regime

Abstract: We consider systems of bosons trapped in a box, in the Gross-Pitaevskii regime. We show that low-energy states exhibit complete Bose-Einstein condensation with an optimal bound on the number of orthogonal excitations, this being a key ingredient to establish the validity of the predictions of Bogoliubov theory in the Gross-Pitaevskii regime.

This talk is based on joint works with C. Boccato, C. Brennecke and B. Schlein.

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Nicolas Rougerie

Title: Bosonic mean-field limits at positive temperature, the 2D case

Abstract: To certain non-linear Schrödinger (NLS) equations, one can associate an invariant Gibbs measure based on the conserved energy. This is the basic ingredient of the Euclidean approach to constructive quantum field theory, as well as the large-time asymptote for the stochastic non-linear heat equation.

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Enno Lenzmann

Title: «Fourier meets Steiner: symmetry results for traveling solitary waves»

Abstract: The aim of this talk is to present definite symmetry results for a general class of nonlinear dispersive PDEs. As a main ingredient, I will discuss sharp rearrangement inequalities for Schwarz and Steiner rearrangements in Fourier space. This is based on joint work with Jérémy Sok.

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Matei Machedon

Title: Uniform in N estimates for a Bosonic system of Hartree-Fock-Bogoliubov type

Abstract: We prove local in time, uniform in N, estimates for the solutions ϕ, Λ and Γ of a coupled system of Hartree-Fock-Bogoliubov type with interaction potential $v_N(x-y) =N^{3 \beta} v(N^{\beta}(x-y))$, with $\beta <1$ and v a Schwartz function (satisfying additional technical requirements). The initial conditions are general functions in a Sobolev-type space, and the expected correlations in Λ develop dynamically in time.

This is joint work with M. Grillakis.

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Peter Pickl

Title: Microscopic Derivation of the Vlasov Equation

Abstract: The microscopic derivation of effective equations for interacting many-body systems is a very active field of research in mathematical physics.

A very old question is the derivation of the Vlasov equation from Newtonian mechanics and results for Lipschitz continuous forces go back to the seventies. In recent years there has been quite a progress in considering singular interaction forces.

In the talk I will present recent results dealing with Coulomb interactions with N-dependent cutoff, where N stands for the number of particles in the system. The cutoff is removed as N tends to infinity and, for very large N will be much smaller than the average distance to the nearest neighbor.

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Chiara Boccato

Title: Bogoliubov Theory in the Gross-Pitaevskii Limit

Abstract: We consider a Bose-Einstein condensate trapped in a box of fixed volume and interacting through a repulsive potential in the Gross-Pitaevskii limit. We prove Bogoliubov?s prediction for the ground state energy and the low-energy excitation spectrum.

This talk is based on joint work with C. Brennecke, S. Cenatiempo and B. Schlein.

Dinh Thi Nguyen

Title: Blow-up of neutron stars in the Hartree-Fock-Bogoliubov theory.

Abstract: It is a fundamental fact that neutron stars collapse when their masses are bigger than a critical number (Chandrasekhar limit). We will study the detail of the collapse phenomenon in the Hartree-Fock-Bogoliubov theory and prove that in the mass critical limit the HFB minimizers develop a universal blow-up profile given by the Lane-Emden solution.

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Alessandro Olgiati

TItle: Ground state energy and BEC for condensate mixtures

Abstract: Condensate mixtures are systems in which two or more different bosonic species are cooled down to a very low temperature and brought to condensation. I will present a rigorous proof of their effective ground state properties.

I will show that, both in the mean field and Gross-Pitaevskii regime, the leading order of the ground state energy is captured by the minimum of a suitable one-body non-linear functional. Moreover, the ground state exhibits Bose-Einstein condensation.

In the mean field regime, by an implementation of Bogoliubov theory, we are also able to compute the next-to-leading order of the ground state energy asymptotics, and to prove a norm approximation for the ground state vector.

All our results hold under a miscibility condition, as is often called in physics literature, that allows us to prove uniqueness of the minimizer of the non-linear theory.

This is a joint work with Alessandro Michelangeli and Phan Th`anh Nam.

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Emanuela Giacomelli

Title: On the Explicit Corner Contribution to Surface Superconductivity

Abstract: In this talk we describe, within the Ginzburg-Landau (GL) theory, the response of a type-II superconducting wire with non-smooth cross section to an external time-independent magnetic field parallel to it.

We focus on the fields whose intensity varies in the so called \textit{surface superconductivity regime}, i.e., superconductivity is confined near the boundary of the sample. A natural question is then \textit{how does the ground state of the GL functional depend on the geometry of the boundary?}

In a preliminary result we show that the presence of corners does not affect the leading order of the energy density. To describe their contribution we then prove a refined energy asymptotics, however this is not enough to have an explicit description. A deeper understanding of the problem in the case of flat angles has led us to conjecture the explicit expression for corner contribution to the energy. Joint work with Michele Correggi.

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Daniele Dimonte

Title: Time Evolution for Condensates in the Thomas-Fermi regime

Abstract: We discuss the time evolution for a many-body system of interacting bosons in the mean-field regime with scaled potential with shrinking support. Under appropriate assumptions the many-body Schrödinger dynamics for N particles is expected to be approximated by a one-particle nonlinear Schrödinger equation. The pure mean-field or GP limits have already been studied in detail, and the approximation proven to be correct on suitable time scales. On the opposite, in spite of its relevance in experimental physics, the Thomas-Fermi (TF) regime has not been fully investigated: in this regime the support of the interaction potential shrinks as the scattering length diverges with N. We show that the dynamic approximation is more subtle in this case, in particular at large time scales, but the effective description is still correct.

Joint work in progress with M. Correggi, D. Mitrouskas and P. Pickl.

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Luca Oddis

Title: Quadratic Forms for 2-Anyon Systems

Abstract: We review the main issues concerning the well-posedness (as suitable self-adjoint operators) of the Hamiltonians of two non-interacting spinless anyons. We show that such operators can be identified with a one-parameter family of self-adjoint extensions of a suitable symmetric operator with Aharonov-Bohm-like magnetic potential. We also derive the explicit expressions of the corresponding quadratic forms and prove their closedness and boundedness from below.

We present a decomposition of the domains of these self-adjoint operators determined by the behaviour of the functions near the coincidence set. We then discuss the extension to the interacting case under suitable assumptions on the potential. Finally, we mention some future perspectives concerning the N-body case. Joint work in progress with M. Correggi

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Søren Mikkelsen

Title: A semiclassical bound on certain commutators

Abstract: In this talk we will for a Schrödinger operator $H_\hbar=-\hbar^2 \Delta +V$ discuss semiclassical commutator bounds of the type:

$$\Vert [A,\boldsymbol{1}_{(-\infty,0]}(H_\hbar)] \Vert_1 \leq C\hbar^{1-d},$$

where A is either the position operator x or the momentum operator $-i\hbar\nabla$. Here C is a positive constant and $\Vert\cdot\Vert_1$ denotes the trace norm. This corresponds to a mean-field version of bounds introduced as an assumption by Benedikter, Porta and Schlein in a study of the mean-field evolution of a fermionic system.

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Matthias Täufer

Title: Homogenization of the control cost for the heat equation: applying mathematical quantum mechanics to control theory

Abstract: We study the controlled heat equation. The game is to appropriately choose an inhomogeneity in the heat equation in such a way that we drive a given initial state to a target state (typically the target state is zero). This inhomogeneity is also called a "control term" and the interesting situation is when this control term is only supported on a subdomain. The control cost is the relation between the minimal norm of such a control term and the norm of the initial state.

In this talk, we will examine what happens to the control cost if we homogenize the subdomain on which the control acts, i.e. if we choose it more and more evenly distributed in space while keeping its overall volume (on bounded domains) or density (on unbounded domains).

Our main workhorse are quantitative, scale-free unique continuation principles, developed in the context of random Schrödinger operators. Thus, this constitutes an example where results from the spectral theory of Schrödinger operators find applications to control theory.

This is joint work with Ivica Nakic, Martin Tautenhahn and Ivan Veselic.

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Edwin Langmann

Title: Heat transport and conformal field theory

Abstract: I will discuss some recent exact result about heat transport in one-dimensional quantum many-body systems obtained with conformal field theory methods.

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Julien Sabin

Title: The semi-classical limit of the Hartree equation at positive density

Abstract: We consider the Hartree equation around a translation-invariant background, and show that the limit of the Wigner transforms of its solutions as the Planck constant goes to zero converge to solutions to the nonlinear Vlasov equation around the classical version of the translation-invariant background. We also discuss the well-posedness of this Vlasov equation at positive density.

This is a joint work with Mathieu Lewin.

Hans Ringström

Title: Linear equations in general relativistic cosmology

Abstract: After a brief introduction to general relativistic cosmology, the talk will focus on linear systems of wave equations on cosmological backgrounds. In particular on the derivation of asymptotics and optimal energy estimates. In the process of proving these estimates, various effective equations arise.

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Marcello Porta

Title: Lower bound on the correlation energy of mean-field fermionic systems

Abstract: In this talk I will discuss the ground state properties of a homogeneous, interacting Fermi gas, in the mean-field regime. In particular, I will focus on the correlation energy, defined as the many-body ground state energy minus the energy of the noninteracting filled Fermi sea. Recently, progress has been made in the understanding the precise value of this quantity, from the point of view of an upper bound. I will report on a lower bound for the correlation energy, which holds for positive, bounded potentials, satisfying a smallness conditions. The bound is optimal in its dependence on the number of particles N, and agrees with the predictions of second order perturbation theory. Joint work with Christian Hainzl and Felix Rexze.

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Niels Benedikter

TItle: Correlation Energy of the Mean-Field Fermi Gas by Collective Bosonization

Abstract: Quantum correlations play an important role in many-body systems; however, their mathematical description is a highly non-trivial task. I explain how correlations in fermionic gases can be described by bosonizing collective pair excitations. This leads us to an effective quadratic bosonic Hamiltonian. We establish a theory of approximate Bogoliubov transformations by which we construct an approximate ground state of the effective Hamiltonian, proving a Gell-Mann--Brueckner type upper bound on the fermionic ground state energy.

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Alessandro Giuliani

Title: Dressed critical curves and quantized Hall conductivity in the Haldane-Hubbard model

Abstract: We consider the Haldane-Hubbard model for electrons on the honeycomb lattice, and prove that for short-range interactions, smaller than the bandwidth, the Hall conductivity is quantized, for all the values of the parameters outside two critical curves. The Hall coefficient remains integer and constant as long as we continuously deform the parameters without crossing the curves; when this happens, the Hall coefficient jumps abruptly to a different integer. The non-renormalization of the Hall conductivity arises as a consequence of lattice conservation laws and of the regularity properties of the current-current correlations. Our method provides a full construction of the critical curves, which are dressed by the electron-electron interaction. The shift of the transition curves manifests itself via apparent infrared divergences in the naive perturbative series, which we resolve via renormalization group methods.

Based on joint works with V. Mastropietro, M. Porta, I. Jauslin.

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Phan Thanh Nam

Title: Bose-Einstein condensation in Gross-Pitaevskii limit: a simple approach

Abstract: We will revisit the convergence rate to the Bose-Einstein condensation in the Gross-Pitaevskii regime, particularly recovering a recent result of Boccato-Brennecke-Cenatiempo-Schlein for the homogeneous gas by a different method.

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Hanne van den Bosch

Title: A critical Poincaré-Sobolev inequality

Abstract: We study a specific Poincaré-Sobolev inequality in bounded domains, that has recently been used to prove a semi-classical bound on the kinetic energy of fermionic many-body states. The corresponding inequality in the entire space is precisely scale invariant and this gives rise to an interesting phenomenon. Optimizers exist for some (most ?) domains and do not exist for some other domains, at least for the isosceles triangle in two dimensions.

In this talk, I will discuss bounds on the constant in the inequality and the proofs of existence and non-existence. This is joint work with Rafael Benguria and Cristóbal Vallejos (PUC, Chile).

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Jeremy Marzuola

Title: Spectral bands, tight binding limits, topological band gaps and bifurcations in periodic Schrödinger operators

Abstract: We will discuss properties of spectral bands for periodic Schrödinger operators established with Keller, Osting and Weinstein in a symmetry setting that includes the Square Lattice and the Lieb Lattice. In particular, we will give conditions for band intersections and describe their geometries.

We will also discuss tight binding limits of these models, as well as modifications thereof that open up band gaps of both topological and non-topological type. In addition, we will discuss current joint work with Bandres, Osting and Rechtsman on nonlinear bifurcations of tight-binding models, as well as nonlinear Dirac models derived in the study of these lattice models. If time permits, we will show some numerical simulations indicating the richness of these models in a variety of settings.

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Christopher Sogge

Title: Resolvent and quasimode estimates on Riemannian manifolds

Abstract: We shall show how the Hadamard parametrix can be sharp used to prove quasimode estimates on compact Riemannian manifolds. Using these techniques along with ones used to study "uniform Sobolev" estimates, we shall also be able to handle Schrödinger operators with singular potentials.

Dirk Hundertmark

Title: The solution of the Gevrey smoothing conjecture for the non-cutoff homogenous Boltzmann equation for Maxwellian molecules

Abstract: It has long been suspected that the non-cutoff Boltzmann operator has similar coercivity properties as a fractional Laplacian. This has led to the hope that the homogenous Boltzmann equation enjoys similar regularity properties as the heat equation with a fractional Laplacian. In particular, the weak solution of the non-cutoff homogenous Boltzmann equation with initial datum in $L^1_2(\mathbb{R}^3)\cap L\log L(\mathbb{R}^3)$, i.e., finite mass, energy and entropy, should immediately become Gevrey regular.

So far, the best available results show that the solution becomes H^{∞} regular for positive times. Gevrey regularity is also known for weak solutions of the linearised Boltzmann equation, where one studies solutions close to a Maxwellian distribution, or under additional decay assumptions on the solutions.

The main problem for establishing Gevrey regularity is that, in order to use the coercivity results on the non-cutoff Boltzmann collision kernel, one has to bound a non-linear and non-local commutator of the Boltzmann kernel with certain sub-exponential weights.

We prove, under the sole assumption that the initial datum is in $L^1_2(\mathbb{R}^3)\cap L\log L(\mathbb{R}^3)$, i.e., finite mass, energy and entropy, that the weak solution of the homogenous Boltzmann becomes Gevrey regular for strictly positive times. The main ingredient in the proof is a new way of estimating the non-local and non-linear commutator.

Joint work with Jean-Marie Barbaroux, Tobias Ried, and Semjon Wugalter.

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Maria Esteban

Title: Magnetic interpolation inequalities in dimensions 2 and 3

Abstract: In this talk I will present various results concerning interpolation inequalities, best constants and information about the extremal functions concerning Schrödinger magnetic operators in dimensions 2 and 3. The particular, and physical interesting cases of constant and of Aharonov-Bohm magnetic fields will be discussed in detail. These works have been made in collaboration with J. Dolbeault, A. Laptev and M. Loss.

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