TITLES AND ABSTRACTS

Mats Andersson:
The $\bar{\partial}$-equation on a non-reduced analytic space.

Abstract: Let X be a pure-dimensional, possibly non-reduced, analytic space. We describe how one can define smooth forms, currents, and the dbar-operator on X. We introduce fine sheaves $\mathcal{A}_X^q$ of $(0, q)$-currents with the following properties: They coincide with the sheaves of smooth forms where the underlying reduced space is smooth and the structure sheaf $\mathcal{O}_X$ is Cohen-Macaulay, and the complex $\mathcal{A}_X^*, \bar{\partial}$ is a (fine) resolution of $\mathcal{O}_X$. This is a joint work with R Lärkäng.

Tien-Cuong Dinh:
Large deviation theorem for zeros of polynomials and random matrices

Abstract: We give abstract versions of the large deviation theorem for the distribution of zeros of polynomials and apply them to the characteristic polynomials of random Hermitian matrices. We obtain new estimates related to the local semi-circular law for the empirical spectral distribution of these matrices when the 4th moments of their entries are controlled. A similar result will be given for random covariance matrices. This talk is based on my paper submitted for a volume in memory of Gennadi Henkin and a forthcoming paper with Duc-Viet VU.

Maciej Dunajski:
Conics and Twistors

Abstract: I shall describe the range of the Radon transform on the space of conics in $\mathbb{CP}^2$, and show that for any function F in this range, the zero locus of F is a four–manifold admitting a scalar–flat Kahler metric which can be constructed explicitly. This is a joint work with Paul Tod.

Michael Eastwood:
Complex methods in real integral geometry

Abstract: There are well-known analogies between holomorphic integral transforms such as the Penrose transform and real integral transforms such as the Radon, Funk, and John transforms. In fact, one can make a precise connection between them and hence use complex methods, as exemplified in the work of Gennadi Henkin, to establish results in the real setting.

Charles Epstein:
Geometry of the Phase Retrieval Problem.

Abstract: Phase retrieval is a problem that arises in a wide range of imaging applications, including x-ray crystallography, x-ray diffraction imaging and ptychography. The data in the phase retrieval problem are samples of the modulus of the Fourier transform of an unknown function. To reconstruct this function one must use auxiliary information to determine the unmeasured Fourier transform phases. There are many algorithms to accomplish task, but none work very well. In this talk we present an analysis of the geometry that underlies these failures and points to new approaches for solving this class of problems.

Jürgen Leiterer:
On the similarity of holomorphic matrices

Absatract: R. Guralnick (Linear Algebra Appl., 85-96, 1988) proved: If two holomorphic matrices on a non-compact connected Riemann surface are locally holomorphically similar, then they are globally holomorphically similar. Actually, he proved a more general theorem for certain Bezout rings, and then applies this to the ring of global holomorphic functions on a non-compact connected Riemann surface. It seems that this proof cannot be generalized to nonsmooth 1-dimensional Stein spaces, and also not to smooth higher dimensional Stein manifolds, because then the ring of global holomorphic functions is not Bezout.

Using other methods, we obtain:

(1) On 1-dimensional Stein spaces, local holomorphic similarity implies global holomorphic similarity. If the space is smooth, then local continuous similarity is sufficient.

(2) On arbitrary Stein spaces, local holomorphic similarity together with global $\mathcal{C}^\infty$ similarity implies global holomorphic similarity.

(3) On contractible 2-dimensional Stein manifolds, local holomorphic similarity implies global holomorphic similarity.

We show by counterexamples:

(1C) For each integer $k \geq 0$, there are irreducible 1-dimensional analytic sets where local $\mathcal{C}^k$-similarity does not imply local holomorphic similarity.

(2C) There exists a Stein domain in $\mathbb{C}^5$ such that, for each integer $k \geq 0$, there is a pair of holomorphic matrices on it, which are locally holomorphically similar, globally C^k-similar, but not globally holomorphically similar.

(3C) There exists a convex domain in $\mathbb{C}^3$ and a pair of holomorphic matrices on it, which are locally holomorphically similar, but not globally holomorphically similar.

Lionel Mason:

The Joukowski transformation, Painleve III and anomalous dimensions

Abstract: Gennadi Henkin was an enthusiast for the use of complex variables methods to solve nonlinear equations. This talk concerns a twistor correspondence based on the Joukowski transformation, that produces solutions to the stationary axisymmetric self-dual Yang-Mills equations. It reduces to give a correspondence for Painleve III and VI and we show how meromorphic solutions impose a quantization of the painleve parameters. The correspondence also underlies a spectral curve for certain anomalous dimensions in quantum field theory which can then be expressed as solutions to the reduced self-dual Yang-Mills equations.

Stefan Nemirovski:

Twistors and contact geometry

Abstract: It was observed by Penrose that the space of light rays of a conformally flat spacetime carries a natural CR structure. In general, however, only the contact structure on that space is canonically defined. It may still be used to relate Lorentz geometry with non-trivial global phenomena - but in contact and symplectic topology rather than in SCV. The talk will survey some results and open problems in this area.

Stephanie Nivoche:

A new proof of a problem of Kolmogorov on ϵ-entropy

Abstract: In the 80's, a Kolmogorov problem about the ϵ-entropy of a class of analytic function was stated :

$$\lim_{\epsilon \to 0}(H_\epsilon(A_K^D)(\ln^{n+1}(1/\epsilon)) = (2C(K,D))((2\pi)^n(n+1)!).$$

In 04, this problem was solved by using technics of pluripotential theory and in particular by proving a Conjecture of Zakharyuta. Here we will present a new proof of Kolmogorov's problem, independently of this conjecture. We will use the asymptotic behaviour of the Bergman kernel of a concentration operator and some properties of special analytic polyhedra.

Roman Novikov:
Inverse scattering in multidimensions

Abstract:We give a review of old and recent results on inverse scattering in multidimensions related with works of G.M. Henkin. This talk is based, in particular, on the following references:

G.M. Henkin, R.G. Novikov, The $\bar{\partial}$-equation in the multidimensional inverse scattering problem, Russ. Math. Surv. 42(3), 109-180, 1987;

G.M. Henkin, N.N Novikova, The reconstruction of the attracting potential in the Sturm-Liouville equation through characteristics of negative discrete spectrum, Stud. Appl. Math. 97, 17-52, 1996;

R.G. Novikov, The $\bar{\partial}$-approach to monochromatic inverse scattering in three dimensions, J. Geom. Anal. 18, 612-631, 2008;

R.G. Novikov, Formulas for phase recovering from phaseless scattering data at fixed frequency, Bull. Sci. Math. 139, 923-936, 2015.

Sergei Pinchuk:
Geometry of proper holomorphic mappings

Abstract: Proper holomorphic mappings between domains in $\mathbb{C}^n (n > 1)$ have been intensively studied since 70th. Gennadi Henkin stood at the origins of this exploration which essentially changed and developed the multidimensional complex analysis. In this talk I will discuss some of the main geometric methods (such as the scaling method, the reflection principle, invariant metrics) and principal results of the theory.

Peter Polyakov:
Explicit Hodge Decomposition on Riemann Surfaces

Abstract: The talk is based on a joint article of the author with Gennadi Henkin (arxiv.org/abs/1507.03272).

Classical Hodge decomposition on the space $Z^{(0,1)}(V) \subset \mathcal{E}^{(0,1)}(V)$ of smooth $\bar{\partial}$-closed $(0,1)$-forms on a smooth algebraic curve $V \subset \mathbb{CP}^n$, $(n \geq 2)$ with metric induced by Fubini-Study metric of $\mathbb{CP}^n$, has the following form

Hodge Theorem . For any form $\phi \in Z^{(0,1)}(V)$ there exists a unique Hodge decomposition:

$$\phi = \bar{\partial} R_1[\phi] + H_1[\phi], \tag{0.1}$$

where H_1 is the orthogonal projection operator from $Z^{(0,1)}(V)$ onto the subspace $\mathcal{H}^{(0,1)}(V)$ of anti-holomorphic $(0,1)$-forms on V, $R_1 = \bar{\partial}^* G_1$, $\bar{\partial}^* : \mathcal{E}^{(0,1)}(V) \to \mathcal{E}^{(0,0)}(V)$, where $\bar{\partial}^* = - * \bar{\partial} *$ is the Hodge dual operator for $\bar{\partial}$, $*$ is the Hodge operator, and G_1 is the Hodge-Green operator for Laplacian $\triangle = \bar{\partial}\bar{\partial}^* + \bar{\partial}^*\bar{\partial}$ on V.

Hodge Theorem was proved by W.V.D. Hodge using the Fredholm's theory of integral equations. The proof was later modified by H. Weyl using his method of orthogonal projection, and by K. Kodaira, who also used the method of orthogonal projection. However, the Hodge Theorem, as it is formulated above, is not explicit enough for some applications. A particular application of an explicit

Hodge-type decomposition that Gennadi had in mind was an explicit solution of the inverse conductivity problem on a bordered Riemann surface, in which the conductivity function has to be reconstructed from the Dirichlet-to-Neumann map on the boundary of the surface (see papers by Calderon [C], and Henkin-Novikov [HN] in a more general setting going back to Gelfand [Ge]).

In [HP] we made the first step toward explicit solution of the inverse conductivity problem by constructing an explicit Hodge-type decomposition for *$\bar{\partial}$-closed residual currents* of homogeneity zero on reduced complete intersections in $\mathbb{CP}^n$. The main results that are discussed in the present talk are some applications of this decomposition for $\bar{\partial}$-closed forms on an arbitrary Riemann surface.

REFERENCES

[C] A.P. Calderon, On an inverse boundary problem, In Seminar on Numerical Analysis and Its Applications to Continuum Physics, Soc. Brasiliera de Matematica, (1980), 61-73.

[Ge] I.M. Gelfand, Some problems of Functional Analysis and Algebra, in Proc. Int. Congr. Math. (Amsterdam 1954), 253-276.

[HN] G.M. Henkin, R. G. Novikov, On the reconstruction of conductivity of a bordered two-dimensional surface in $\mathbb{R}^3$ from electrical current measurements, on its boundary, J. Geom. Anal. 21 (2011), no. 3, 543-587.

[HP] G.M. Henkin, P.L. Polyakov, Explicit Hodge-type decomposition on projective complete intersections, The Journal of Geometric Analysis, 26(1), 2016.

Berit Stensønes:
Non simply connected regions of attraction for Automorphisms.

Abstract: We shall show how to construct an region of attraction in $\mathbb{C}^2$ which is biholomorphic to $\mathbb{C} \times \mathbb{C}^*$ and also show some ideas for finding such domains of higher connectivity in higher dimensions.

Alex Tumanov:
Symplectic non-squeezing in infinite dimension

Abstract: The celebrated Gromov's non-squeezing theorem of 1985 says that the unit ball in a symplectic space can be symplectically embedded in the circular cylinder only if the radius of the cylinder is at least 1. Hamiltonian differential equations provide examples of symplectic transformations in infinite dimension. Known results on the non-squeezing property in Hilbert spaces cover compact perturbations of linear symplectic transformations and several specific non-linear PDEs, including the periodic Korteweg - de Vries equation and the periodic cubic Schrödinger equation. We present a version of the non-squeezing theorem for Hilbert spaces. We apply the result to the discrete nonlinear Schrödinger equation. This work is joint with Alexander Sukhov.

Nils Øvrelid:
A new approach to localized Sobolev estimates in complex analysis

Abstract: The standard approach to Sobolev estimates near the boundary for d-bar problems is the method of elliptic regularization of J. J. Kohn and Louis Nirenberg (1965). The problem is that one does not a priori know that the solution is sufficiently regular that the operations necessary to derive the desired estimates are justified. One therefore makes small perturbations of the $\bar{\partial}$- Neumann problem that are elliptic boundary value problems, for which one has optimal regularity results, prove estimates uniform wrt. perturbations, and pass to a weak limit. For this, certain global assumptions about the domain D are necessary. We introduce a purely local approach, minimizing global assumptions. In a neighbourhood U of a smooth part S of the boundary of D, we replace the tangential pseudodifferential operators A used by K. -N. by sequences of 0th order operators A_j converging

weakly to A. For these, the necessary commutations and integrations by parts are clearly justified, and we derive estimates uniform in j, from which the desired tangential Sobolev estimates simply follow from the monotone convergence theorem. In these arguments, the $\bar\partial$- Neumann boundary conditions only need to be satisfied on S.This allows us to derive localized Sobolev estimates in U for the minimal solution of the $\bar\partial$- equation, when it exists, and for the Bergman projection, under the weakest possible assumptions on D. We also show that the boundary singularities at S of these objects are locally determined. Our results are most complete when S consists of subelliptic points; but we have also some results when we allow points of infinite type.