Abstracts: Lefschetz Properties in Algebra Geometry and Combinatorics, at Institut Mittag Leffler

July 10-14, 2017

Nancy Abdallah (Lindköping University)

"Combinatorial Invariance of KLV Polynomials for Fixed Points Free Involutions"

Abstract: Let S_{2n} be the symmetric group of permutation of $\{1, \cdots, 2n\}$, and $F_{2n} \subset S_{2n}$ be the set of fixed points free involutions. To every interval $[u, v]$ in the poset F_{2n} ordered by the Bruhat order, we associate a KLV-polynomial $P_{u,v}$. Using a combinatorial concept called Special Partial Matching or SPM, we prove that these polynomials are combinatorially invariant for upper intervals, i.e. for intervals $[u, w_0]$ where w_0 is the maximal element of F_{2n}. This gives a generalization of the combinatorial invariance of the classical Kazhdan-Lusztig polynomials. This is a joint work with Axel Hultman.

Karim Adiprasito (The Hebrew University of Jerusalem)

"$T < 4E$"

Abstract: I will discuss a new approach to proving the hard Lefschetz theorem for non-projective toric varieties, and apply this technique to solve some problems in combinatorics. Specifically, I will prove that a 2-dimensional simplicial complex embedding in R^4 on E edges and T triangles satisfies the inequality $T < 4E$.

(talk Wed or Thursday)

Nasrim Altafi (KTH Royal Institute of Technology)

"Lefschetz property of equigenerated monomial ideals"

Abstract: Let $S = k[x_1, \ldots, x_n]$ be the polynomial ring over the field k with characteristic zero and maximal ideal $\mathfrak{m} = (x_1, \ldots, x_n)$.
In this talk we will study the Lefschetz properties of $\mathfrak{m}$-primary monomial ideals generated in degree d. I will give a sharp upper bound for the number of generators of such ideals which do not satisfy the Weak Lefschetz property. In fact, the multiplication map by linear form on

S/I from degree $d-1$ to degree d is not surjective.
In addition, I will give a proof of the following conjecture by D. Eisenbud, C. Huneke and B. Ulrich in the case when I is a monomial ideal and its minimal free resolution is linear for $n-1$ steps.

Conjecture 0.1. Suppose $I \subset S$ is $\mathfrak{m}$-primary ideal generated in degree d and its minimal free resolution is linear for $p-1$ steps then

$$\mathfrak{m}^d \subset I + (l_p, \ldots, l_n)^2$$

for sufficiently general linear forms $l_i's$.

Rodrigo Gondim (Universidade Federal Rural de Pernambuco)

"On mixed Hessians and the Lefschetz properties"

Abstract We review the notion of Hessian of higher order introduced by Maeno-Watanabe to study the Strong Lefschetz properties and we give a generalization of this notion. We introduce a Mixed Dual Hessian matrix and we show that it is the matrix of the multiplication map by a power of a linear form $\bullet L^{j-i} : A_i \to A_j$. In this way this matrix controls SLP and WLP, furthermore, it is compatible with compositions. We give some applications of this construction. It is part of a work in progress together with Giuseppe Zappalá (U. of Cantania).

Shihoko Ishii (Tokyo Women's Christian University)

"Jet closures and the local isomorphism problem"

Abstract: In the talk, I will introduce the local isomorphism problem which is reduced to the case that the scheme is the spectrum of an Artin local ring. For every variety X and a point $x \in X$, we can associate the local m-jet schemes $X_m(x)$ for $m \in \mathbb{N}$. If the two singularities (X, x) and (Y, y) are isomorphic, then the local m-jet schemes $X_m(x)$ and $Y_m(y)$ are isomorphic for every $m \in \mathbb{N}$. The local isomorphism problem asks the converse: if the local jet schemes $X_m(x)$ and $Y_m(y)$ are isomorphic for every $m \in \mathbb{N}$, then are the singularities (X, x) and (Y, y) isomorphic? This is not true if X is a non-Noetherian scheme. But for the Noetherian case, at the moment, we do not have either any counter example nor a proof of the problem. If X is a scheme of finite type over an algebraically closed field k, then the problem is reduced to the case that X is the spectrum of an Artinian complete intersection local ring of embedding dimension 2.

Martina Juhnke-Kubitzke (University of Osnäbruck)

"Lower bound theorems for balanced manifolds"

Abstract: A simplicial complex of dimension $d-1$ is said to be balanced if its graph is d-colorable. We prove an analogue of the generalized lower bound theorem for balanced triangulations of closed homology manifolds and balanced triangulations of orientable homology manifolds with boundary under an additional assumption that all proper links of these triangulations have the weak Lefschetz property. As a consequence, we show that if Δ is an arbitrary balanced triangulation of any closed homology manifold of dimension $d-1 \geq 3$, then

$$2h_2(\Delta) - (d-1)h_1(\Delta) \geq 4\binom{d}{2}(\tilde{\beta}_1(\Delta) - \tilde{\beta}_0(\Delta)).$$

This is joint work with Satoshi Murai, Isabella Novik and Connor Sawaske.

Leila Khatami (Union College, Schenectady)

"Equations for loci of commuting nilpotent matrices"

Abstract: We discuss recent progress in determining the loci of nilpotent matrices of given Jordan type, whose maximal commuting nilpotent Jordan type has two parts.

This is joint work with Mats Boij, Tony Iarrobino, and Bart Van Steirteghem.

Samuel Lundqvist (Stockholm University)

"The Weak Lefschetz property of some monomial complete intersections" (arXiv:1612.00411)

Abstract: Let $S = \mathbb{C}[x_1, \ldots, x_n]/I^k$, where I is the ideal generated by the d-th powers of the variables. When does S have the WLP? When $k = 1$, it is well known that S has the SLP. When $k > 1$, things are more involved. Partial answers are given in a joint work with Mats Boij and Ralf Fröberg. I will give a summary of the results and will then focus on some open problems: When $n = 3$, we have a conjecture on the pairs (d, k) such that S has the WLP. The conjecture is settled in the failing cases only. When $n > 3$ and $(d, k) = (2, 2)$, we believe that S has the WLP (and the SLP), but we have only been able to prove that S has the WLP when n is odd. When $n > 12$, we believe that S fails the WLP unless $(d, k) = (2, 2)$, but have no results in this direction (besides computer calculations).

Chris McDaniel (Endicott College)

"A GKM description of the equivariant coinvariant ring of a finite reflection group"

Abstract: The equivariant cohomology ring of a flag manifold can be described in terms of the invariant theory of the associated Weyl group–this is the so-called equivariant coinvariant ring. GKM (Goresky-Kottwitz-MacPherson) theory gives an alternative description of the equivariant cohomology ring in terms the torus orbit structure on the flag manifold. In this talk I will describe an algebraic analogue of GKM theory for the equivariant coinvariant ring of an arbitrary finite reflection group.

Emilia Mezzetti (University of Trieste)

"Togliatti systems and Galois coverings" (arXiv:1611.05620)

Abstract: We study the homogeneous Artinian ideals of the polynomial ring $K[x, y, z]$ generated by the homogenous polynomials of degree d which are invariant under an action of the cyclic group $\mathbb{Z}/d\mathbb{Z}$ for any $d \geq 3$. We prove that every such ideal is a monomial Togliatti system, i.e. an Artinian ideal generated by homogeneous polynomials of degree d, failing the Weak Lefshetz Property in degree $d - 1$. Moreover they are minimal if the action is defined by a diagonal matrix having on the diagonal $(1, e, e^a)$, where e is a primitive d-th root of the unity. We get a complete description when d is prime or a power of a prime. We also establish the relation of these systems with linear Ceva configurations. This is joint work with Rosa Maria Mirò-Roig, originated from work in Banff ("Lefschetz Properties and Artinian Algebras", BIRS March 2016).

Juan Migliore (University of Notre Dame)

"Results and problems on Lefschetz properties for ideals of powers of linear forms"

Abstract: Let R be a polynomial ring in r variables. Let L be a general linear form. A result of Stanley and of Watanabe says that if I is an ideal generated by r powers of linearly independent linear forms (e.g. the variables), and if j and k are any positive integers, then multiplication by L^k from the degree j component of R/I to the degree $j + k$ component has maximal rank (i.e. the strong Lefschetz property holds). This leads to the question of what happens when the ideal is generated by powers of more than r linear forms. Over the last decade or so several papers have addressed different aspects of this problem, most often focusing on the case $k = 1$ (i.e. studying the weak Lefschetz property). In the last half year or so, progress has been made on the problem when $k > 1$. I will give an overview of the history of this problem, talk about recent work in collaboration with R. Miró-Roig (2016), with U. Nagel (2017), and with C. Huneke and U. Nagel (in preparation). I will also mention some open problems and directions for new research.

Satoshi Murai (Osaka University)

"Lefschetz properties of balanced 3-polytopes"

Abstract A $(d?1)$-dimensional simplicial complex is said to be balanced if its graph is d-colorable. If a simplicial complex is balanced, then its StanleyReisner ring has a special system of parameters induced by the coloring. We prove that the Artinian reduction of the StanleyReisner ring of a balanced simplicial 3-polytope with respect to this special system of parameters has the strong Lefschetz property if the characteristic of the base field is not two or three. Moreover, we characterize $(2, 1)$-balanced simplicial polytopes, i.e., polytopes with exactly one red vertex and two blue vertices in each facet, such that an analogous property holds. This is a joint work with David Cook, Martina Juhnke-Kubitze, and Eran Nevo begun at BIRS "Lefschetz Properties" 2016.

Lisa Niklasson (Stockholm University)

"The weak Lefschetz property of monomial complete intersections in positive characteristic"

Abstract: There is a classification of the monomial complete intersections in positive characteristic, which possess the strong Lefschetz property. For the weak Lefschetz property, there is not yet a complete classification, but there are several partial results, using different techniques. In this talk we will discuss what is known (and not) about this problem.

Hal Schenck (University of Illinois at Urbana Champagne)

"The Weak Lefschetz property for quotients by Quadratic Monomials"

Abstract: Michałek–Miró-Roig recently gave a beautiful geometric characterization of Artinian quotients by ideals generated by quadratic or cubic monomials, such that the multiplication map by a general linear form fails to be injective in the first nontrivial degree. Their work was motivated by conjectures of Ilardi and Mezzetti-Miró-Roig-Ottaviani connecting the failure to Laplace equations and classical results of Togliatti on osculating planes. We investigate WLP for quotients by quadratic monomial ideals, explaining failure of the Weak Lefschetz Property for some cases not covered by the results of Michałek–Miró-Roig. Joint work with J. Migliore and U. Nagel.

Larry Smith (University of Göttingen)

"The Algebra/Geometry Dictionary and a Conjecture of Bertram Kostant"

Abstract: Fix a ground field F and suppose that $G \subset \mathrm{GL}(n, \mathsf{F})$ is a finite subgroup. Set $V = \mathsf{F}^{\mathsf{n}}$ and suppose that $U \leq V$ is a linear subspace setwise stabilized by a subgroup H of G. Then a G-invariant polynomial on V restricts to an H-invariant polynomial on U. We are interested in knowing if this map is a surjection, meaning onto set theoretically, starting from the assumption that the induced map of orbit spaces $U/H \longrightarrow V/G$ is a monomorphism. The Algebra $\leftrightarrow$ Geometry dictionary tells us that the map of invariant algebras is therefore an epimorphism in the category of reduced algebraic sets; BUT does that mean it is a surjection? The answer turns out to be quite interesting and I will discuss what I have learned in the past year concerning this situation. Time permitting I will also discuss the Kostant Conjecture and how what I have learned applies to it. In some sense this is a continuation of the talk I gave at BIRS.

Jean Vallès (Université de Pau)

"Bundles associated to line arrangement and Lefschetz Properties"

Abstract: We will consider the relationship between line arrangements $\{l_1, \dots, l_n\}$ in $\mathbb{P}^2$, linear systems $(l_1^r, \dots, l_n^r)$ failing the SLP and unexpected curves (introduced by Cook-Harbourne-Migliore-Nagel). More precisely, in order to describe this relationship, we will associate to a given set of points $Z \subset \check{\mathbb{P}}^2$ a rank $d+1$ vector bundle $\mathcal{T}_Z^{(d)} := p_* q^* \mathcal{I}_Z(d)$ (where p and q are the left and right projections from the incidence variety $\{(x,l) \in \mathbb{P}^2 \times \check{\mathbb{P}}^2 | x \in l\}$); we are particularly interested in finite sets Z such that the splitting of $\mathcal{T}_Z^{(d)}$ on a general line is not balanced. Indeed, a non balanced splitting of $\mathcal{T}_Z^{(d)}$ leads eventually to an ideal $(l_1^r, \dots, l_n^r)$ that fails the SLP and to unexpected curves.

In a second step, we will extend the notion of free arrangements to the rank $d+1$ bundles $\mathcal{T}_Z^{(d)}$. We will give examples of Z such that $\mathcal{T}_Z^{(d)} = \mathcal{O}_{\mathbb{P}^2}(-a_1) \oplus \cdots \oplus \mathcal{O}_{\mathbb{P}^2}(-a_{d+1})$ and we will discuss the link between freeness of $\mathcal{T}_Z^{(1)}$ and freeness of $\mathcal{T}_Z^{(d)}$.

Akihito Wachi (Hokkaido University of Education)

"A characterization of the Macaulay dual generators for quadratic complete intersections"

Abstract: Let $F \in R = K[x_1, \dots, x_n]$ be a homogeneous polynomial of degree d. The quotient ring A of R by the annihilator of F is a graded artinian Gorenstein K-algebra, and F is called the Macaulay dual generator for A. If B is a subalgebra of A which is Gorenstein with the same socle degree as A, we describe the Macaulay dual generator for B in terms of F. Furthermore when $n = d$, we give necessary and sufficient conditions on the polynomial F for A to be a complete intersection.

Junzo Watanabe (Tokai University)

'The Weyr form seems a better tool for Artinian algebras than the Jordan form"

Abstract: After an introduction to Weyr forms, I discuss the Weyr structure for the n-th Sierpinski matrix. Then this is used to prove that the algebra $K[x_1, \dots x_n]/(x_1^t, \dots, x_n^t)$ has the strong Lefschetz property.

The Sierpinski matrix is defined inductively:
$B_0 = 1, B_1 = ((11), (01)), \dots, B_{n+1} = ((B_n B_n), (O, B_n))$.