# Branching problems for reductive groups

Mittag-Leffler Institute, May 16 - 20, 2016

### Monday May 16th

**09:30 – 10:20:** George McNinch (Tufts University)

*An overview of representations of reductive algebraic groups*

**11:00 – 11:50:** Donna Testerman (École Polytechnique Fédérale de Lausanne)

*Simple algebraic groups: restricting representations to subgroups, results and techniques*

**16:30 – 17:10:** Irina Suprunenko (National Academy of Sciences of Belarus)

*Big composition factors in restrictions of modular representations of classical algebraic groups to subsystem subgroups*

*Abstract.* The goal of this talk is to discuss the construction of composition factors with certain special properties in restrictions of modular irreducible representations of classical algebraic groups to subsystem subgroups with two simple components. We shall deal with factors that are in a certain sense big enough (or not too small) for both components of a subgroup under consideration. The existence of such factors yield effective tools for solving a number of questions, in particular, for finding or estimating various parameters of the images of individual elements in representations of such groups, and not only for elements of relevant subsystem subgroups.

In this talk a subsystem subgroup of a simple algebraic group is a subgroup generated by all root subgroups associated with the roots of a fixed subsystem of the root system. We consider the following types of subsystem subgroups:

$$\begin{array}{rr} A_m(K)\times A_n(K)\subset A_{m+n+1}(K), & SL_{m+1}(K)\times SL_{n+1}(K)\subset SL_{m+n+2}(K),\\ B_m(K)\times D_n(K)\subset B_{m+n}(K), & SO_{2m+1}(K)\times SO_{2n}(K)\subset SO_{2(m+n)+1}(K),\\ C_m(K)\times C_n(K)\subset C_{m+n}(K), & Sp_{2m}(K)\times Sp_{2n}(K)\subset Sp_{2(m+n)}(K),\\ D_m(K)\times D_n(K)\subset D_{m+n}(K), & SO_{2m}(K)\times SO_{2n}(K)\subset SO_{2(n+m)}(K)\end{array}$$

In all these cases relevant subsystem subgroups are maximal subsystem subgroups, the right column shows these subgroups in the standard realization of a group.

For an irreducible representation $\varphi$ of a classical algebraic group denote by $s(\varphi)$ the value of its highest weight on the maximal root. For the restriction of $\varphi$ to a subsystem subgroup with two simple components $H_1$ and $H_2$ of type mentioned above, we find a composition factor of the form $\varphi_1\otimes\varphi_2$ where $\varphi_i$ is an irreducible representation of $H_i$, $s(\varphi_1)=s(\varphi)$, and $s(\varphi_2)$ is as big as possible. For $p$-restricted representations in positive characteristic $p$ with a large highest weight with respect to $p$, we want to find a factor where $\varphi_1$ has a large highest weight with respect to $p$ and $s(\varphi_1)$ and $s(\varphi_2)$ are not very far from the maximal possible values. Under some minor restrictions on $m$, such factors are constructed for almost all $p$-restricted representations of special linear and symplectic groups with $s(\varphi)\geq p$. The factors obtained can be used for estimating the number of the Jordan blocks of the maximal size in the images of certain unipotent elements in irreducible representations of relevant groups.

**17:20 – 17:40:** Anna Osinovskaya (National Academy of Sciences of Belarus)

*The restrictions of representations of the special linear group to subsystem subgroups of type $A_2$*

**17:50 – 18:20:** Mikaël Cavallin (TU Kaiserslautern)

*On the structure of certain Weyl modules for the special orthogonal group*

*Abstract.* Let $V$ be a finite-dimensional vector space over an algebraically closed field $K$ having characteristic $p \geq 0$. In this talk, we show how the natural embedding of $X = \mathrm{SO}(V)$ in $Y = \mathrm{SL}(V)$ can be used in order to determine the structure of certain Weyl modules for $X$.

## Tuesday May 17th

**09:30 – 10:20:** Sasha Kleshchev (University of Oregon)

*Restrictions in non-defining characteristic*

**11:00 – 11:50:** Gerhard Hiss (RWTH Aachen)

*Maximal subgroups of finite classical groups and imprimitive representations of finite quasisimple groups*

**16:30 – 17:10:** David Stewart (Newcastle University)

*Some new results unbounding cohomology*

**17:20 – 17:50:** Rolf Källstrom (University of Gävle)

*Decomposition of modules over invariant differential operators*

*Abstract.* Given a finite subgroup $G$ of the general linear group $GL(V)$ we have the invariant ring $A = B^G$ in the symmetric algebra $B = S(V)$, $D_B$ the ring of differential operators on $B$, and $D = D_B^G$ its subring of invariant differential operators. We assume here that $V$ is a finite-dimensional vector space over a field of characteristic 0. There is a correspondence between the category of finite-dimensional representations of $G$ and the semisimple category of $D$-modules generated by the simples in $B$, regarded as $D$-module, so that representations of $G$ can be studied geometrically by systems of differential equations. Moreover, we show that $D$-modules in this category can be decomposed using a natural lowest weight method, much in analogy with, say, the BGG category $\mathcal{O}$. This gives a natural understanding of branching structures in certain situations, where one for instance recovers the Young basis and the branching rules for the symmetric group using lowest weight arguments; it would be interesting to find more examples. I will give a non-technical overview.

## Wednesday May 18th

**09:30 – 10:20:** Colva Roney-Dougal (University of St Andrews)

*Maximal subgroups of low rank almost simple groups*

**11:00 – 11:50:** Pham Tiep (University of Arizona)

*Low-dimensional representations of finite quasisimple groups: A survey*

*Abstract.* We will survey known results on low-dimensional representations of finite quasisimple groups. We will also discuss some recent applications of these results.

## Thursday May 19th

**09:30 – 10:20:** Kay Magaard (University of Birmingham)

*Maximal subgroups of classical groups: open problems*

**11:00 – 11:50:** Frank Lübeck (RWTH Aachen)

*Computing explicit examples of* $\dim H^1(G, V)$

**16:30 – 17:00:** Adam Thomas (University of Bristol)

*Complete reducibility in good (and bad) characteristic*

**17:10 – 17:50:** David Craven (University of Birmingham)

*Subspace stabilizers and infinite subgroups*

*Abstract.* A result of Liebeck and Seitz states that, if $x$ is a semisimple element in a simple algebraic group $G$, then there exists an explicit constant, $t(G)$ such that, if $x$ has order greater than $t(G)$, then $x$ is contained inside an infinite subgroup of $G$ stabilizing the same subspaces of the adjoint module $L(G)$. This result is used to give bounds on potential maximal subgroups of exceptional groups of Lie type. In this talk we give some improvements to the bound of $t(G)$ by swapping $L(G)$ for the minimal module, when $G$ is exceptional and not of type $E_8$.

## Friday May 20th

**09:30 – 10:00:** Frank Himstedt (TU Munich)

*On characters of parabolic subgroups of finite groups of Lie type of small rank*

*Abstract.* In my talk I give an overview over my work (with coauthors) on characters of parabolic subgroups of some series of finite groups of Lie type of small rank. In particular, I describe how information on the restriction of irreducible characters of $SO_7(q)$ and $Sp_6(q)$ to a maximal parabolic subgroup led to the determination of some decomposition numbers of these groups in non-defining characteristic (this part is joint work with Felix Noeske).

**10:10 – 10:30:** Amanda Schaeffer Fry (Metropolitan State University of Denver)

*On the irreducible restrictions from* $\mathrm{SO}_7(q)$ *to* $G_2(q)$

**11:10 – 11:50:** Martin Liebeck (Imperial College)

*Multiplicity-free representations of algebraic groups*