Applied Mathematics

Towards invariant domain preserving high order continuous finite elements for conservation laws

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Galerkin finite element approximation is known to be unstable for numerical approximation of convection dominated problems, especially nonlinear conservation laws. To overcome this drawback, a mesh-dependent, consistent numerical stabilization is usually added. However, this arises new questions on a proper choice for stabilization parameters and mesh-sizes, the computational cost, the convergence to the right physical solution, and the so-called invariant domain property. For scalar conservation laws, the notion of invariant domain property is closely related to the traditional maximum principle.

In this talk we will present a stabilized explicit continuous finite element method that satisfies invariant domain property for (system of) conservation laws for arbitrary unstructured meshes. The new method is formally first order and does not require any a priori knowledge of quantities like local wavespeed, proportionality constant, or local mesh-size. We prove that the method converges to the unique entropy solution for scalar conservation laws in any space dimensions.

The method can be made high order in time by using Strong Stability Preserving Runge-Kutta methods. In this talk, we discuss the main ingredients that are needed for achieving high order accuracy in space. A series of benchmarks for scalar and system of conservation laws will be presented.

This is joint work with Jean-Luc Guermond and Bojan Popov.