Dynamical Systems

Instability of degenerate elliptic equilibria in Hamiltonian dynamics

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KAM theory asserts that generically an elliptic fixed point of a Hamiltonian system is stable in a probabilistic sense, or KAM stable: the fixed point is accumulated by a positive measure set of invariant Lagrangian tori. It was conjectured by M. Herman in his ICM98 lecture that for analytic Hamiltonians, KAM stability holds in a neighborhood of an elliptic fixed point if its frequency vector is assumed to be Diophantine. The conjecture is known to be true in two degrees of freedom, but remains open in general. Partial results in this direction were recently obtained Eliasson, Fayad and Krikorian.

Below analytic regularity, Herman proved that KAM stability of a Diophantine equilibrium holds without any twist condition in C^∞ in 2 degrees of freedom. In his ICM98 lecture Herman announced that KAM stabil-ity of Diophantine equilibria does not hold for smooth Hamiltonians in 4 or more degrees of freedom, without giving any clew about the possible counter-examples. He also wrote that nothing was known about KAM stability of Diophantine equilibria for smooth Hamiltonians in 3 degrees of freedom.

In this talk, which is based on a joint work with Bassam Fayad, we settle this problem by presenting examples of smooth Hamiltonians for any $d \geq 3$ having non KAM stable elliptic equilibria with arbitrary frequency.