Contents

LONG TALKS (50 MINUTES)

Kenneth Falconer | University of St Andrews

Projections of Fractals - Old and New. — Marstrand's theorems from the 1950s on the dimension of orthogonal projections of fractals are central in fractal geometry. This talk will discuss some of the many generalisations and variations on Marstrand's results that have emerged over the years, for example different notions of dimensions, exceptional directions, special cases such as self-similar sets, self-affine sets, and percolation sets, and will include a number of recent results.

De-Jun Feng | Chinese University of Hong Kong

Dimension of invariant measures for affine iterated function systems. — In this talk, we investigate the local dimensions of certain fractal measures. We prove the exact dimensionality of ergodic invariant measures for every contractive affine iterated function systems. These measures are the push-forwards of the ergodic invariant measures in the symbolic space under the coding map, and include all the self-affine measures. We also establish the Ledrappier-Young type dimension formula. Furthermore, the result extends to average contracting affine IFSs. This completes several previous results. Some applications are given to the dimension of self-affine sets and measures.

Jonathan Fraser | University of St Andrews

The Assouad spectrum. — The Assouad dimension is a familiar notion of dimension which, for a given metric space, returns the infimal exponent $\alpha \geq 0$ such that for any pair of scales $0 < r < R$, any ball of radius R may be covered by a constant times $(R/r)^\alpha$ balls of radius r. Motivated by this, we introduce a spectrum of dimensions designed to yield more information about the scaling structure of the space. More precisely, to each $\theta \in (0, 1)$, we associate the appropriate analogue of the Assouad dimension with the restriction that the two scales r and R used in the definition satisfy $\log R/\log r = \theta$. We discuss the resulting 'dimension spectrum' (as a function of θ) including its basic analytic and geometric properties as well as the precise calculation of the spectrum for some specific examples. This is joint work with Han Yu (St Andrews).

Maarit Järvenpää | University of Oulu

Random covering sets. — Limsup sets, defined as upper limits of various sequences of sets, play an important role in different areas of mathematics. Random covering sets are a class of limsup sets defined by means of a family of randomly distributed subsets of the d-dimensional torus. We discuss various problems related to random covering sets with special emphasis to their dimensional properties.

Tamás Keleti | Eötvös Loránd University

Baire category arguments in geometric measure theory. — The Baire category theorem states that in a complete metric space (X, d) the intersection of countably many dense open sets is non-empty, or equivalently dense. It is said that a property "typically holds" in (X, d) if it holds for every point of a dense G-delta set. Often it is an efficient way to show that some objects with given properties exist by showing that these properties hold typically in an appropriate complete metric space. For example, it is not hard to show that a typical continuous function on $[0, 1]$ is nowhere differentiable and nowhere monotone.

In this talk we will see applications of this method in geometric measure theory. We will study the dimensions of a typical compact subset of a Euclidean space. We will also see how Tom Körner constructed Besicovitch sets of Lebesgue measure zero in the plane using Baire category argument. I also plan to explain how we could use with Alan Chang, Marianna Csörnyei and Kornélia Héra this method to construct other small sets that contain a large collection of objects.

Sabrina Kombrink | University of Lübeck

Asymptotic expansions of renewal functions with applications to Steiner formula for fractals. — Classical renewal theorems provide the leading asymptotic term of renewal functions. They have been widely used in mathematics and other disciplines. Here, we are concerned with an extension which also provides the lower asymptotic terms of certain renewal functions. This expansion is applied to obtain a fractal analogue of the famous Steiner formula.

The Steiner formula for a non-empty compact convex subset K of d-dimensional Euclidean space states that the volume of the epsilon-parallel set of K can be expressed as a polynomial in epsilon of degree d. The coefficients of the polynomial carry important information on the geometry of the convex set, such as the volume, the surface area and the Euler characteristic. For fractal sets the epsilon-parallel volume is more involved and cannot be written as a polynomial in epsilon. In this talk we discuss the behaviour of the epsilon-parallel volume of certain fractals and analogues of the Steiner formula generated by our extended renewal theorem. Moreover, we explore the geometric information encoded in the analogues of the exponents and coefficients.

Michał Rams | Polish Academy of Sciences

Lyapunov spectrum for matrix cocycles. — Given a finite family of $SL(2, \mathbb{R})$ matrices $(A_i)_{i \in \{1,\dots M\}}$, we consider for every $\omega \in \{1, \dots, M\}^{\mathbb{N}}$ the limit

$$\chi(\omega) = \lim_{n\to\infty} \frac{1}{n} \log ||A_{\omega_n} \cdot \ldots \cdot A_{\omega_1}||$$

(whenever the limit exists), which we call *Lyapunov exponent* of the sequence ω. Our goal is to describe the set $L_a = \{\omega \in \{1, \dots, M\}^{\mathbb{N}}; \chi(\omega) = a\}$ in terms of topological entropy. This is achieved as a consequence of a more general statement about skew products of circle homeomorphisms. It is a joint work with Lorenzo Diaz and Katrin Gelfert.

Károly Simon | Budapest University of Technology and Economics

Singularity of self-similar measures (joint work with Lajos Vágó (Budapest)). — One-parameter families of self-similar measures are consid-ered in the special case when the similarity dimension of each measure in the family is greater than one.

In addition to the family of infinite Bernoulli convolution measures, another natural example of such a family is obtained as follows: Take the natural (uniformly distributed) measure on the Sierpinski carpet and consider its orthogonal projections to the angle-α line for every $\alpha \in (0, \pi)$. (Here α is the parameter.)

A recent result of Shmerkin and Solomyak says that these measures are absolutely continuous for all but a set of zero Hausdorff dimension of parameters.

With Lajos Vágó, we considered the same kind of one-parameter families of self-similar measures, but we asked what happens with the absolute continuity of the measures for parameters which are typical in the topological sense (dense and G_δ). The surprising partial results we obtained are the topic of the talk.

SHORT TALKS (25 MINUTES)

Jason Atnip | University of North Texas

Bowen's Formula for Non-Autonomous Graph Directed Markov Systems. — In his 1979 paper Rufus Bowen showed that the Hausdorff dimension for the limit set of a quasi-Fuchsian group is given by the unique zero of the pressure function. In this talk, we develop the theory of non-autonomous graph directed Markov systems, which generalize iterated function systems, and then prove a Bowen's formula result concerning the dimension of the limit set. This theory can then be applied to give a lower bound for the Hausdorff dimension of the set of escaping points for many classes of transcendental functions.

Catherine Bruce | University of Manchester

Projections of Gibbs measures on self-conformal sets in the plane. — Hochman and Shmerkin used Furstenburg's theory of CP-processes to prove strong Marstrand results for self-similar sets and measures with dense rotations which satisfy the open set condition. That is, to prove that the Hausdorff dimension of the projections of such sets and measures is the maximum possible value for every projection. Here we extend such a result to Gibbs measures on self-conformal sets in the plane without requiring any separation condition.

Stuart Burrell | University of St Andrews

Inhomogeneous attractors and upper box dimension. — In 1985, Barnsley [1] introduced the notion of an inhomogeneous attractor for an iterated function system (IFS). These consist of the standard (homogeneous) attractor and all images of some some compact set C via composition of maps from the IFS. For the case where the maps in the IFS are similarities, Fraser [2] established bounds for the upper box dimension based on the solution to Hutchinson's formula and the dimensions of the homogeneous attractor and condensation set. We first give an overview of this work, and then describe current work to generalise this for an IFS consisting of conformal mappings.

[1] M. F. Barnsley and S. Demko. Iterated function systems and the global construction of fractals, Proc. R. Soc. Lond. Ser. A, 399, (1985), 243–275.

[2] Jonathan Fraser, 'Inhomogeneous self-similar sets and box dimensions', Studia Mathematica, 213:133-156 (2012).

Changhao Chen | University of New South Wales

Salem sets in vector spaces over finite fields. — We first talking about Salem sets in Euclidean spaces. Informally speaking, a set in Euclidean space is called a Salem set if there exist measures on this set, and the Fourier transform of these measures have "optimal decay". Secondly, we show the finite fields version of Salem sets. The talk based on the speaker's recent project: Salem sets in vector spaces over finite fields.

Douglas Howroyd | University of St Andrews

Some dimension results on iterated sumsets. — Sumsets have been studied for many years in several different contexts, in particular the dimension of sumsets has often been considered. Thanks to the recent inverse theorem of Hochman we can study the limiting behaviour of the upper box and Assouad dimensions of iterated sumsets, with self-similar sets providing particularly nice examples. This is joint work with Jonathan Fraser and Han Yu.

Malo Jézéquel | École Normale Supérieure de Paris

Linear response and periodic points for expanding maps of the circle of finite differentiability. — We will present the notion of linear response for expanding maps of the circle and then give a formula for it. This formula was proposed by Pollicott and Vytnova in the analytic setting and is in this context particularly efficient considering numerical calculation. However, we shall see that this formula still holds under some assumptions of finite differentiability. The main tool is a result of parameter regularity for some dynamical determinants.

Gabriella Keszthelyi | Eötvös Loránd University

Equi-topological entropy curves for skew tent maps in the square (Joint work with Zoltan Buczolich). — We consider skew tent maps $T_{\alpha,\beta}(x)$ such that $(\alpha,\beta) \in [0,1]^2$ is the turning point of $T_{\alpha,\beta}$, that is, $T_{\alpha,\beta} = \frac{\beta}{\alpha}x$ for $0 \le x \le \alpha$ and $T_{\alpha,\beta}(x) = \frac{\beta}{1-\alpha}(1-x)$ for $\alpha < x \le 1$. We denote by $\underline{M} = K(\alpha,\beta)$ the kneading sequence of $T(\alpha,\beta)$ and by $h(\alpha,\beta)$ its topological entropy. For a given kneading sequence $\underline{M}$ we consider equi-kneading, (or equi-topological entropy, or isentrope) curves $(\alpha, \varphi_{\underline{M}}(\alpha))$ such that $K(\alpha, \varphi_{\underline{M}}(\alpha)) = \underline{M}$. To study the behavior of these curves an auxiliary function $\Theta_{\underline{M}}(\alpha,\beta)$ is introduced. For this function $\Theta_{\underline{M}}(\alpha, \varphi_{\underline{M}}(\alpha)) = 0$, but it may happen that for some kneading sequences $\Theta_{\underline{M}}(\alpha,\beta) = 0$ for some $\beta < \varphi_{\underline{M}}(\alpha)$ with (α,β) still in the dynamically interesting quarter of the unit square. Using $\Theta_{\underline{M}}$ we show that the curves $(\alpha, \varphi_{\underline{M}}(\alpha))$ hit the diagonal $\{(\beta,\beta) : 0.5 < \beta < 1\}$ almost perpendicularly if (β,β) is close to $(1,1)$. Answering a question asked by M. Misiurewicz at a conference we show that these curves are not necessarily exactly orthogonal to the diagonal, for example for $\underline{M} = RLLRC$ the curve $(\alpha, \varphi_{\underline{M}}(\alpha))$ is not orthogonal to the diagonal. On the other hand, for $\underline{M} = RLC$ it is.
With different parametrization properties of equi-kneading maps for skew tent maps were considered by J.C. Marcuard, M. Misiurewicz and E. Visinescu.

Istvan Kolossvary | Budapest University of Technology and Economics

Triangular Gatzouras Lalley-type planar carpets. — We construct a family of planar self-affine carpets using (lower) triangular matrices in a way that generalizes the original Gatzouras Lalley carpets defined by diagonal matrices. For typical parameters, which can be explicitly checked, we prove that the inequalities between the Hausdorff, box and affinity dimension of the attractor are strict even under the rectangular open set condition. The dimension formulas coincide with those of the Gatzouras Lalley carpets. Furthermore, we show sufficient conditions under which overlaps in the construction, satisfying a sort of transversality condition, do not affect the dimension. We give examples to illustrate the results, including a collection of self-affine continuous curves which contain an orientation reversing mapping.

Lawrence Lee | University of St Andrews

Diophantine approximation on manifolds and lower bounds for Hausdorff dimension. — Diophantine approximation is a branch of analytic number theory that aims to understand how well real numbers may be approximated by rationals. In this talk I will provide a brief introduction to the area and I will show how we can establish analogues of some classical theorems from the real line on manifolds, using the Mass Transference Principle of Beresnevich and Velani. This is based on joint work with Victor Beresnevich (York), Robert Vaughan (Penn State) and Sanju Velani (York).

Marco López | University of North Texas

Dimension of shrinking targets arising from nonautonomous systems. — For a dynamical system on a metric space X we define a shrinking target set consisting of those points in X whose orbits hit a ball of shrinking radius innitely often. In special cases, such sets arise from Diophantine approximation. One aspect of such sets that is often studied is their Hausdorff dimension. We will talk about how thermodynamic formalism can be applied to characterize the Hausdorff dimension of such sets for a certain class of nonautonomous iterated function systems.

Balázs Maga | Eötvös Loránd University

Baire categorical aspects of first passage percolation. — In the previous decades, the theory of first passage percolation became a highly important area of probability theory. In this work, we will observe what can be said about the corresponding structure if we forget about the probability measure defined on the product space of edges and simply consider topology in the terms of residuality. We focus on interesting questions arising in the probabilistic setup that make sense in this setting, too. We will see that certain classical almost sure events, as the existence of geodesics have residual counterparts, while the notion of the limit shape or time constants gets as chaotic as possible.

Antoine Marnat | University of York

Transference between homogeneous and inhomogeneous Diophantine approximation on manifolds. — We provide an extension of the transference results of Beresnevich and Velani connecting homogeneous and inhomogeneous Diophantine approximation on manifolds and provide bounds for inhomogeneous Diophantine exponents of affine subspaces and their non degenerate sub manifolds.

Arne Mosbach | University of Bremen

Positive definite Dirac combs of mixing interval maps. — Through the work of Baake, Damanik, Haynes, Lenz and Moody, to name but a few, the theory of diffraction has received much attention in resent years. One of the main questions being, if the diffraction gained from a dirac comb is again discrete, which is then called pure point, absolute continuous or singular continuous? Indeed, there is a large framework for sets generated by regular cut and project schemes (CPS), whose diffraction turns out to be pure point.

In this talk, we will discuss the diffraction of Dirac combs generated by certain mixing transformations on the unit interval. In contrast to the typical setting of regular CPS, it turns out that the diffraction of our dynamically defined Dirac comb is the sum of a Dirac mass at zero and an absolute continuous measure.

Felipe Perez Pereira | University of Bristol

Hausdorff dimension of Bernoulli measures with infinite entropy. — It is known that for finite branched maps of the interval, under certain conditions the Hausdorff dimension of an invariant measure μ can be computed by the formula $\dim_H \mu = h_\mu/\lambda_\mu$, where h_μ and λ_μ are the entropy and Lyapunov exponent of μ respectively. This formula can be extended to the case with countably many branches if the entropy is supposed to be finite. In this talk we calculate the dimension of measures with infinite entropy for certain maps, emphasizing the differences with the finite entropy case.

Alessandro Pezzoni | University of York

A Hausdorff measure result for a problem of approximation by polynomials. — The study of the set $L_n(w)$ of points $x \in \mathbb{R}$ for which there are infinitely many integer polynomials of degree up to n such that

$$|P(x)| < H(P)^{-w}$$

has a long history, which dates back to 1932 with Kurt Mahler. In this talk we will introduce this problem in the context of Metric Number Theory and we shall show how to determine the generalised Hausdorff measure of $L_3(w)$ for a given dimension function g. Time permitting, we will also discuss how one might try to do this for $n > 3$.

Ruxi Shi | University of Picardie Jules Verne

Fuglede conjecture on the field of p-adic numbers. — For a locally compact abelian group G, Fuglede conjecture states that a Borel set is spectral if and only if it tiles the group G by translation. In the case $G = \mathbb{R}^n$, it have been studied for long time since Fuglede formulated this conjecture in 1974. It is proved to be false for $n \geq 3$ but it is still open for $n = 1, 2$. In this talk, I will consider the case $G = Q_p$ the field of p-adic numbers and give an affirmative answer to this conjecture. This is the joint work with A.Fan, S.Fan and L.Liao.

Gáspár Vértesy | Eötvös Loránd University

Some properties of Okamoto's functions. — In 2005 Hisashi Okamoto defined a family of parametrized continuous self-affine functions: $g_s : [0,1] \to [0,1]$, where $s \in (0,1)$ is the parameter. This was a generalization of some other maps such as the Cantor function. Okamoto's maps have several interesting properties (e.g. they are like typical continuous functions in many senses including differentiability and local monotonicity). He studied differentiability in his article. We examine some other properties of these functions.

For all $s \in (0,1)$ and α real number we determine whether g_s is Hölder-α. We also look at the local behaviour of the functions: we analyse the sets where the functions are not locally monotone.

Finally, we show an algorithm, which can be used for estimating the upper and lower box dimension of the level sets if $s = 2/3$ (this method is extendable for all $s \geq 2/3$ with a small restriction).

Yufeng Wu | Chinese University of Hong Kong

Matrix representations for some self-similar and self-affine measures on $\mathbb{R}^d$. — We establish matrix representations for self-similar and self-affine measures on $\mathbb{R}^d$ generated by IFSs satisfying the finite type condition. As an application, we prove that for a self-similar measure μ on $\mathbb{R}^d$ generated by an IFS which has commensurable contraction ratios and satisfies the finite type condition, the L_q-spectrum of μ is differentiable on $(0,\infty)$. This extends an earlier result of Feng to higher dimensions.

Lixuan Zheng | University Paris-Est

The exceptional sets on the run-length function of beta-expansions. — Let $\beta > 1$ and the run-length function $r_n(x,\beta)$ be the maximal length of consecutive zeros amongst the first n digits in the β-expansion of $x \in [0,1]$. The exceptional set

$$E_{\max}^{\varphi} = \left\{ x \in [0,1] : \liminf_{n\to\infty} \frac{r_n(x,\beta)}{\varphi(n)} = 0, \limsup_{n\to\infty} \frac{r_n(x,\beta)}{\varphi(n)} = +\infty \right\}$$

is investigated, where $\varphi : \mathbb{N} \to \mathbb{R}^+$ is a monotonically increasing function with $\lim_{n\to\infty} \varphi(n) = +\infty$. We prove that the set $E_{\max}^{\varphi}$ is either empty or of full Hausdorff dimension and residual in $[0,1]$ according to the increasing rate of φ .

Monday 4th – Friday 8th, December 2017

Workshop on Fractals and Dimensions, Institut Mittag-Leffler