Logic

Independence, the Continuum Hypothesis, and the nature of infinity

Hugh Woodin
Department of Mathematics, University of California, Berkeley, USA

The modern mathematical story of infinity began in the period 1879-84 with a series of papers by Cantor that defined the fundamental framework of the subject. Within 40 years the key ZFC axioms for Set Theory were in place and the stage was set for the detailed development of transfinite mathematics, or so it seemed. However, in a completely unexpected development, Cohen showed in 1963 that even the most basic problem of Set Theory, that of Cantor's Continuum Hypothesis, was not solvable on the basis of the ZFC axioms.

The 50 years since Cohen's work has seen a vast development of Cohen's method and the realization that the occurrence of unsolvable problems is ubiquitous in Set Theory. This arguably challenges the very conception of Cantor on which Set Theory is based.

Thus a fundamental dilemma has emerged. On the one hand, the discovery, also over the last 50 years, of a rich hierarchy of axioms of infinity seems to argue that Cantor's conception is fundamentally sound. But on the other hand, the developments of Cohen's method over this same period seem to strongly suggest there can be no preferred extension of the ZFC axioms to a system of axioms that can escape the ramifications of Cohen's method.

But this dilemma was itself based on a misconception and recent discoveries suggest there is a resolution.