

Logic

Descriptive Set Theory of Topological Equivalence Relations

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Borel reducibility equips the set of all equivalence relations (ER) on Borel subsets of Polish spaces with a partial order structure. Since many mathematical object classes (countable models, separable Banach spaces, C^* -algebras and Polish spaces themselves) can be coded as Borel subsets of Polish spaces, this theory provides tools for the analysis of the complexity of classification problems. In this talk I will present some recent results on topological ERs: A knot is an embedding of a circle into R^3 and two knots are equivalent if there is a homeomorphism of R^3 taking one to the other. This ER is induced by a Polish group action, but is not classifiable by countable structures. The homeomorphism relation on subsets of R^3 which are the union of an open set and a point is strictly above the universal ER induced by a Polish group action (UERPG). This is in a sense best possible, because the homeomorphism on locally compact spaces has exactly the same complexity as UERPG. A major part of the latter result is due to Sabok, Zielinski, Kechris and Solecki. I conclude with a list of open questions in this area.