Nonlinear Partial Differential Equations
Boundary value problems and equations arising in fluid mechanics

Modulated elliptic wave and a train of asymptotic solitons in a vicinity of the leading edge for MKdV

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We discuss the behavior of the solution of the modified Korteweg-de Vries equation of a step type in a vicinity of the leading edge.

The initial data tends to zero as $x \to +\infty$ and to some positive constant c as $x \to -\infty$. In 1989 E. Khruslov and V. Kotlyarov have found that for a large time the solution breaks up into a train of asymptotic solitons located in the domain $4c^2t - C_N \log t < x < 4c^2t$ (C_N is a constant). The number N of these solitons grows unboundedly as $t \to +\infty$.

The behavior of the solution in the domain slightly apart from the leading edge $(-6c^2 + \varepsilon)t < x < (4c^2 - \varepsilon)t$ was studied in [3], and this solution is described by a modulated elliptic wave.

It would be natural to assume that the modulated elliptic asymptotics is still valid in the domain $(4c^2 - \varepsilon)t < x < 4c^2t$, but in [4] it was shown that the modulated elliptic wave and the train of asymptotic solitons do not coincide in the domain $4c^2t - C_N \log t < x < 4c^2t$ because of a phase shift. This indicates a possible presence of a further transition region in the already small domain $(4c^2 - \varepsilon)t < x < 4c^2t - C_N \log t$.

References

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