Nonlinear Partial Differential Equations
Boundary value problems and equations arising in fluid mechanics

On some geodesic flows on Fréchet–Lie groups

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It is the aim of this talk to present a geometric method to study various model equations appearing in mathematical hydrodynamics as geodesic flows of right-invariant metrics induced by suitable Fourier multipliers on the Fréchet-Lie group of all diffeomorphisms of the n-dimensional torus and the Euclidean n-space. This approach covers in particular right-invariant metrics induced by Sobolev norms of fractional order. It is shown that the corresponding initial value problem is well-posed in the smooth category and that the Riemannian exponential mapping is a smooth local diffeomorphism, provided that the symbol complies with certain mild structural conditions. If the order of the inertia operator is larger than 3 then the induced weak Riemannian metric is geodesically complete. Applications to the equations of Camassa–Holm, Degasperis–Procesi, and Constantin–Lax–Majda are also discussed.

References

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