

NORDAN 2016: Several Complex Variables

Degrees of polynomial approximation in holomorphic Carleman classes

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In this talk, we extend results of M.S. Baouendi and C. Goulaouic (Ann. Inst. Fourier, 1971 ; Trans. Amer. Math. Soc, 1974), obtained for compacts of \mathbb{R}^N with analytic boundary. If K is a compact of $\mathbb{C}^N \simeq \mathbb{R}^{2N}$, ($N \geq 1$), $\mathcal{H}_M(K)$ is the space of ∂ -Whitney jets on K which are of class $\{M\}$, where $M(t) = t^t e^{t\mu(t)}$, $t \gg 0$ and μ belongs to a Hardy field. We prove that a jet $F := (F^\alpha)_{\alpha \in \mathbb{N}^{2N}} \in \mathcal{H}_M(K)$ if and only if there exists a constant $C > 0$, such that

$$\lim_{n \rightarrow \infty} d_n(F^\alpha, K) \exp(C\bar{\omega}_{K,M}(n)) = 0, \quad \text{for all } \alpha \in \mathbb{N}^{2N},$$

where $d_n(\cdot, K)$ is the distance, for the uniform norm on K to the complex vectorial space of polynomials of degree at most n , and where $\bar{\omega}_{K,M}$ is a weight depending on the class $\{M\}$ and K . If K is Whitney-regular

$$\mathcal{H}_M(K) \simeq \{f \in \mathcal{E}^\infty(K) \cap \mathcal{O}(K) : \\ \exists C > 0, \exists \rho > 0, \|D^\alpha f\|_K \leq C\rho^{|\alpha|} M(|\alpha|), (\forall \alpha \in \mathbb{N}^N)\}.$$

In this situation, $f \in \mathcal{H}_M(K)$ if and only if $\lim_{n \rightarrow \infty} d_n(f, K) e^{C\bar{\omega}(n)} = 0$, where $C > 0$ and $\bar{\omega}$ is a weight depending on $\{M\}$. Finally, we announce similar results in the situation where K is a compact of some Stein manifold. A crucial role is played by a new geometric criteria: the Lojasiewicz-Siciak condition for the Green function of K . This is a joint work with Boutayeb El Ammari and Laurent P. Gendre.