Number theory

Bad reduction of curves with CM jacobians

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An abelian variety defined over a number field and having complex multiplication (CM) has potentially good reduction everywhere. If a curve of positive genus which is defined over a number field has good reduction at a given finite place, then so does its jacobian variety. However, the converse statement is false already in the genus 2 case, as can be seen in the entry $[I_0 - I_0 - m]$ in Namikawa and Ueno's classification table of fibres in pencils of curves of genus 2. In this joint work with Philipp Habegger, our main result states that this phenomenon prevails for certain families of curves.

We prove the following result: Let F be a real quadratic number field. Up to isomorphisms there are only finitely many curves C of genus 2 defined over $\overline{\mathbb{Q}}$ with good reduction everywhere and such that the jacobian $Jac(C)$ has CM by the maximal order of a quartic, cyclic, totally imaginary number field containing F. Hence such a curve will almost always have stable bad reduction at some prime whereas its jacobian has good reduction everywhere. A remark is that one can exhibit an infinite family of genus 2 curves with CM jacobian such that the endomorphism ring is the ring of algebraic integers in a cyclic extension of $\mathbb{Q}$ of degree 4 that contains $\mathbb{Q}(\sqrt{5})$.