Number theory

Serre weights and wild ramification

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Serre's Conjecture (proved by Khare and Wintenberger) asserts that every odd, irreducible representation from $\mathrm{Gal}(K/\mathbb{Q})$ to $GL_2(\mathbb{F}_{p^r})$ arises from a modular form; furthermore Serre specifies the minimal weight and level of a form giving rise to the Galois representation. The recipe for the weight depends on the local behavior of the representation at the prime p, and this dependence becomes much more subtle in generalizations of Serre's Conjecture. In particular, when $\mathbb{Q}$ is replaced by a totally real field, work of Gee and others determines the weights of Hilbert modular forms giving rise to a mod p Galois representation in terms of the existence of crystalline lifts, but the dependence of the weights on wild ramification is not explicit. I'll discuss joint work with Dembele and Roberts towards making it so, or equivalently, describing wild ramification in reductions of two-dimensional crystalline Galois representations.