Number theory

Harmonic weak Siegel Maass forms

Martin Westerholt-Raum
Chalmers, Sweden

Harmonic weak Maass forms for $SL(2,\mathbb{R})$ where defined by Bruinier and Funke more than ten years ago. They have been successfully applied to combinatorial problems thanks to their overlap with indefinite theta functions. Their genuine arithmetic, on the other hand, is linked to derivatives of L-series, as demonstrated by Bruinier and Ono. We start by revisiting these connections.

Siegel modular forms are modular forms for the group $Sp(g,\mathbb{R})$. If $g = 1$ they coincide with elliptic modular forms, but in the general case they are more intricate. In joint work with Bringmann and Richter, the speaker studied real analytic Siegel modular forms and connected their Fourier Jacobi coefficients in the case of $g = 2$ to harmonic weak Maass forms for $SL(2,\mathbb{R})$. We showcase this connection, and exhibit the importance of Fourier Jacobi coefficients in the study of Siegel modular forms.

Finally, we discuss the existence of harmonic weak Siegel Maass forms. Using the connection of Eisenstein series and principal series representations, one manages to obtain sufficiently tight control of Dolbeault cohomology to show that every non-holomorphic Saito-Kurokawa lift can be further lifted to a harmonic weak Siegel Maass form. We discuss potential applications to derivatives of L-series.