Numerical Analysis and PDE

Compressive space-time Galerkin discretizations of parabolic partial differential equations

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We study linear parabolic initial-value problems in a space-time variational formulation based on fractional calculus. This formulation uses "time derivatives of order one half" in time. We show that for linear, parabolic initial-boundary value problems on $(0, \infty)$, the corresponding bilinear form admits an inf-sup condition with sparse tensor product trial and test function spaces. We deduce optimality of compressive, space-time Galerkin discretizations, where stability of Galerkin approximations is implied by the well-posedness of the parabolic operator equation. The variational setting adopted here ad-mits more general Riesz bases than previous work; in particular, no stability in negative order Sobolev spaces on the spatial or temporal domains is re-quired of the Riesz bases accommodated by the present formulation. The trial and test spaces are based on Sobolev spaces of equal order $1/2$ with respect to the temporal variable. Sparse tensor products of multi-level de-compositions of the spatial and temporal spaces in Galerkin discretizations lead to large, non-symmetric linear systems of equations. We prove that their condition numbers are uniformly bounded with respect to the discretization level. In terms of the total number of degrees of freedom, the convergence orders equal, up to logarithmic terms, those of best N-term approximations of solutions of the corresponding elliptic problems.
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