Operator Theory and Analytic Function Spaces

Harmonic maps and shift-invariant subspaces

Alexandru Aleman
Lund University, Sweden

The talk concerns harmonic maps (critical points of energy functionals) from a simply connected Riemann surface into the unitary group $U(n)$. Following the ideas of K. Uhlenbeck and Segal, such maps can be studied with help of an infinite dimensional vector bundle of closed subspaces of $L^2(\mathbb{T}, \mathbb{C}^n)$, which are invariant for the forward shift. This approach leads to an interesting interplay between geometric objects and properties of invariant subspaces. The purpose is is to emphasize some analytic questions which arise in this context. The material is based on joint work with M.J. Martin, A.M. Persson, M. Svensson and J. Wood.

Coulomb gas ensembles in 2D

Håkan Hedenmalm
KTH Royal Institute of Technology, Sweden

Coulomb gas in 2D consists of repelling particles confined by an external potential. We study the condensation of the particle ensemble in the many particles limit with condensation to a compact subset known as the droplet.

Bloch functions and asymptotic tail variance

Håkan Hedenmalm
KTH Royal Institute of Technology, Sweden

We introduce a new concept, the tail variance, associated with a Gauss-like distribution. We apply this in a setting of Bloch functions, and obtain a sharp exponential square integrability result which resembles Beurling's theorem as well as the John-Nirenberg theorem. The theorem is applied to give a strong universal estimate for the quasiconformal integral mean spectrum $B(k, t)$.

Affine and linear invariant families of harmonic mappings

María Martín
University of Eastern Finland

We establish a generalization of Pommerenke's result that gives the order of the family of holomorphic mappings of bounded Schwarzian norm by deriving the analogous result for harmonic mappings. Our result shows consistency between the conjectured value for the order of the class S_H and one natural candidate for being extremal for the Schwarzian norm. Joint work with M. Chuaqui and R. Hernández.

Basis properties of generalised p-cosine functions

Houry Melkonian
Heriot-Watt University, UK

Consider a periodic function F, such that its restriction to the unit segment lies in the Banach space $L_s = L_s(0, 1)$ for $s > 1$. Denote by S the family of dilations $F(nx)$ for all n positive integer. The purpose of this talk is to discuss the following question: When does S form a Schauder basis of L_s?

At first sight, one might think that this question has been studied considerably in the past. For instance in the context of Paley-Wiener-type theorems. As it turns, this has not been the case, and the latter does not seem to be of much use in this respect.

We will formulate general criteria which apply to the particular case of F being the p-sine and the p-cosine functions. Both these functions arise naturally in the context of the non-linear eigenvalue problem associated to the one-dimensional p-Laplacian in the unit segment. Our main goal will be to determine a range of values for the parameter p, such that the dilated p-cosine functions form a Schauder basis of L_s. Our results improve upon those from [Edmunds, Gurka, Lang, *J. Math. Anal. Appl.* 420 (2014)].

Dynamics of the Gauss maps and the Hilbert transform

Alfonso Montes-Rodríguez
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A pair (Γ, Λ), where $\Gamma \subset \mathbb{R}^2$ is a locally rectifiable curve and $\Lambda \subset \mathbb{R}^2$ is a *Heisenberg uniqueness pair* if an absolutely continuous finite complex-valued Borel measure supported on Γ whose Fourier transform vanishes on Λ necessarily is the zero measure. Here, absolute continuity is with respect to arc length measure. Recently, it was shown by Hedenmalm and Montes that if Γ is the hyperbola $x_1 x_2 = M^2/(4\pi^2)$ (where $M > 0$ is the mass), and Λ is the lattice-cross $(\alpha\mathbb{Z} \times \{0\}) \cup (\{0\} \times \beta\mathbb{Z})$, where α, β are positive reals, then (Γ, Λ) is a Heisenberg uniqueness pair if and only if $\alpha\beta M^2 \leq 4\pi^2$. The Fourier transform of a measure supported on a hyperbola solves the one-dimensional Klein-Gordon equation, so the theorem supplies very thin uniqueness sets for a class of solutions to this equation. By rescaling, we may assume that the mass equals $M = 2\pi$, and then the above-mentioned theorem is equivalent to the following assertion: *the functions*

$$\mathrm{e}^{\mathrm{i}\pi\alpha m t}, \quad \mathrm{e}^{-\mathrm{i}\pi\beta n/t}, \qquad m, n \in \mathbb{Z},$$

span a weak-star dense subspace of $L^\infty(\mathbb{R})$ if and only if $0 < \alpha\beta \leq 1$. The proof involved ideas from Ergodic Theory. To be more specific, in the critical regime $\alpha\beta = 1$, the crucial fact was that the Gauss-type map $t \mapsto -1/t$ modulo $2\mathbb{Z}$ on $[-1, 1]$ has an ergodic absolutely continuous invariant measure with infinite total mass. However, the case of the semi-axis $\mathbb{R}_+$ as well as the holomorphic counterpart remained open. We completely solve these two problems. Both results can be stated in terms of Heisenberg uniqueness, but here, we prefer the concrete formulation. As for the semi-axis, *we can show that the restriction to* $\mathbb{R}_+$ *of the functions*

$$\mathrm{e}^{\mathrm{i}\pi\alpha m t}, \quad \mathrm{e}^{-\mathrm{i}\pi\beta n/t}, \qquad m, n \in \mathbb{Z},$$

span a weak-star dense subspace of $L^\infty(\mathbb{R}_+)$ *if and only if* $0 < \alpha\beta \leq 4$. In the critical regime $\alpha\beta = 4$, the weak-star span misses the mark by one dimension only. The proof of this statement is based on the ergodic properties of the standard Gauss map $t \mapsto 1/t$ mod $\mathbb{Z}$ on the interval $[0, 1]$. In particular, we find that for $1 < \alpha\beta < 4$, there exist nontrivial functions $f \in L^1(\mathbb{R})$ with

$$\int_{\mathbb{R}} \mathrm{e}^{\mathrm{i}\pi\alpha m t} f(t)\mathrm{d}t = \int_{\mathbb{R}} \mathrm{e}^{-\mathrm{i}\pi\beta n/t} f(t)\mathrm{d}t = 0, \qquad m, n \in \mathbb{Z},$$

and that each such function is uniquely determined by its restriction to any of the semiaxes $\mathbb{R}_+$ and $\mathbb{R}_-$. This is an instance of *dynamical unique continuation.*

As for the holomorphic counterpart, *we show that the functions*

$$e^{i\pi\alpha mt}, \quad e^{-i\pi\beta n/t}, \qquad m, n \in \mathbb{Z}_+ \cup \{0\},$$

span a weak-star dense subspace of $H^\infty_+(\mathbb{R})$ *if and only if* $0 < \alpha\beta \leq 1$. Here, $H^\infty_+(\mathbb{R})$ is the subspace of $L^\infty(\mathbb{R})$ which consists of those functions whose Poisson extensions to the upper half-plane are holomorphic. In the critical regime $\alpha\beta = 1$, the proof relies on the nonexistence of a certain invariant distribution in the predual of real H^∞ for the above-mentioned Gauss-type map on the interval $](1, 1)$, which is a new result of dynamical flavor. To attain it, we need to handle in a subtle way series of powers of transfer operators, a rather intractable problem where even the recent advances by Melbourne and Terhesiu do not apply. More specifically, our approach – which is obtained by combining ideas from Ergodic Theory with ideas from Harmonic Analysis – involves a splitting of the Hilbert kernel, as induced by the transfer operator. The careful analysis of this splitting involves detors to the Hurwitz zeta function as well as to the theory of totally positive matrices.

Joint work with Håkan Hedenmalm, KTH.

Duality of weighted Bergman spaces with small exponents

Antti Perälä
University of Eastern Finland

We consider the weighted Bergman spaces consisting of p-integrable analytic functions for small p. Under general conditions on the weight, we show that the duals of such Bergman spaces are all isomorphic to the Bloch space. The dual pairing is induced by another weight, which has to be chosen quite carefully. The result is well-known in the setting of standard weights; and we explain the main obstructions one has to overcome in this more general case.

This talk is based on a joint work with Jouni Rättyä.

Singular numbers of composition operators on Hardy or Bergman spaces of the polydisk

Hervé Queffélec
Université Lille Nord de France

Let $\mathbb{D}$ be the unit disk, and $\Omega = \mathbb{D}^d$ or $\mathbb{D}^\infty \cap \ell^1$, and let $\varphi : \Omega \to \Omega$ be an analytic map, inducing a composition operator C_φ, formally

$$C_\varphi(f) = f \circ \varphi.$$

We are interested in the action of C_φ on Hardy or Bergman spaces $H^2(\Omega)$ or $B^2(\Omega)$, in particular in its compactness, its membership in Schatten classes, its singular (or approximation) numbers. The case $d = 1$ begins to be pretty well understood. We will present the first results obtained in the multivariate case $d \geq 2$; we will show in particular that the decay of approximation numbers becomes slower and slower when d increases. The infinite-dimensional case will also be touched. This is joint work with F. Bayart, D. Li, and L. Rodriguez-Piazza.

Radial average operator and Bergman projection

Jouni Rättyä
University of Eastern Finland

Many problems concerning derivatives of inner functions in weighted Bergman spaces are related to the question of when one may apply the Schwarz-Pick lemma inside the Bergman norm integral without any essential loss of information. The radial doubling weights having this property are those for which a certain radial average operator is bounded from the weighted Bergman space to the corresponding Lebesgue space. The boundedness of this average operator can be described by a kind of Bekolle-Bonami- or Muckenhoupt-type condition, and is in turn equivalent to the boundedness of the Bergman projection in an appropriate setting. This average operator works as a model for the study of the boundedness of the Bergman projection, induced by a radial doubling weight, acting from one Lebesgue space to another, both being induced by radial regular weights. This talk is about the above-mentioned results.

Large GCD sums and extreme values of the Riemann zeta function

Kristian Seip
Norwegian University of Science and Technology

We prove that for every c, $0 < c < 1/\sqrt{2}$, there exists a β, $0 < \beta < 1$, such that the maximum of $|\zeta(1/2 + it)|$ on the interval $T^\beta \le t \le T$ exceeds $\exp\left(c\sqrt{\log T \log\log\log T / \log\log T}\right)$ for all T large enough. Our proof uses Soundararajan's resonance method and a special multiplicative function arising from our study of certain GCD sums. This is joint work with Andriy Bondarenko.

Solid hulls of weighted Banach spaces of entire functions

Jari Taskinen
University of Helsinki, Finland

Since it is often impossible to describe a non-Hilbert Banach space of analytic functions on the disc or the plane in terms of the Taylor coefficients, the next best thing is to find the solid hull of the given space. This means, rougly, finding the strongest growth condition that the coefficients of the functions in the given space have to satisfy. We characterize the solid hulls of a large class of Banach spaces of entire functions, which are endowed with weighted sup-norms $\|f\|_v = \sup v(z)|f(z)|$, where the weight $v(z)$ is radial, continuous, decreasing as $|z| \to \infty$. For example for $v(z) = \exp(-|z|)$, we show that the solid hull consists exactly on those sequences (b_m), for which

$$\sup_n \sum_{m=n^2+1}^{(n+1)^2} |b_m|^2 e^{-2n^2} n^{4m} < \infty$$

The solid hull of the corresponding spaces on the disc, with doubling weights, were found by Bennet, Stegena, Timoney in 1981, and we develop their methods in connection with some more recent techniques on weighted H^∞ spaces.

This is joint work with José Bonet.

Polynomial lemniscates and their fingerprints: from analysis to topology

Alexander Vasiliev
University of Bergen, Norway

Our main objects of study are shapes given by polynomial lemniscates, and their fingerprints. After giving some simple relations between analytic properties of shapes and fingerprints, we focus on the inflection points of fingerprints, their number and geometric meaning. Furthermore, we study dynamics of zeros of lemniscate-generic polynomials and their explosion that occur by planting singularities at certain moment, and then performing their versal deformation by a braid group. We call this dynamics polynomial fireworks and show that it is realized by construction of a braid operad.

This is joint work with Dmitry Khavinson (Tampa).