PDE session

L^2 Solvability of boundary value problems for divergence form parabolic equations with complex coefficients

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We consider parabolic operators of the form

$$\partial_t + \mathcal{L},\ \mathcal{L} = -\operatorname{div} A(X,t)\nabla,$$

in $\mathbb{R}^{n+2}_+ := \{(X,t) = (x, x_{n+1}, t) \in \mathbb{R}^n \times \mathbb{R} \times \mathbb{R} :\ x_{n+1} > 0\}$, $n \geq 1$. We assume that A is a $(n+1) \times (n+1)$-dimensional matrix which is bounded, measurable, uniformly elliptic and complex, and we assume, in addition, that the entries of A are independent of the spatial coordinate x_{n+1} as well as of the time coordinate t. For such operators we prove that the boundedness and invertibility of the corresponding layer potential operators are stable on $L^2(\mathbb{R}^{n+1}, \mathbb{C}) = L^2(\partial\mathbb{R}^{n+2}_+, \mathbb{C})$ under complex, L^∞ perturbations of the coefficient matrix. Subsequently, using this general result, we establish solvability of the Dirichlet, Neumann and Regularity problems for $\partial_t + \mathcal{L}$, by way of layer potentials and with data in L^2, assuming that the coefficient matrix is a small complex perturbation of either a constant matrix or of a real and symmetric matrix.