Applications of the multivariate tail process for extremal inference

Anja Janssen
University of Copenhagen, Denmark

Multivariate regularly varying time series are a common tool for modelling the dynamics of heavy-tailed processes of dimension larger than one. Let $(X_t)_{t\in\mathbb{Z}}$ be a stationary d-dimensional regularly varying process. The extremal behavior of this process can be described by the index $\alpha > 0$ of regular variation and the law of the so-called spectral tail process $(\Theta_t)_{t\in\mathbb{Z}}$, for which

$$\mathcal{L}\left(\frac{X_{-n}}{x},\ldots,\frac{X_m}{x}\,\middle|\,\|X_0\|>x\right) \overset{w}{\Rightarrow} \mathcal{L}(Y\cdot\Theta_{-n},\ldots,Y\cdot\Theta_m), \quad x\to\infty,$$

with a Pareto(α)-distributed random variable Y which is independent of $(\Theta_t)_{t\in\mathbb{Z}}$, cf. Basrak & Segers (2009). The spectral tail process satisfies a certain property which is sometimes called the "time change formula" that describes its behavior when shifted in time, cf. Basrak & Segers (2009). We are interested in estimating the law of Θ_t for $t\in\mathbb{Z}$ with a focus on the cases $t=0,1$. These two quantities are of interest in particular for Markov processes $(X_t)_{t\in\mathbb{Z}}$ where their joint distribution (together with the value of α) already determines the whole distribution of $(\Theta_t)_{t\in\mathbb{Z}}$, cf. Janßen and Segers (2014). By extending an idea used in Drees, Segers & Warchoł (2015) from the univariate to the multivariate case we show that it may be helpful for the estimation of Θ_1 to make use of the time change formula that gives us

$$P(\Theta_1\in A) = E\left(\mathbf{1}_A\left(\frac{\Theta_0}{\|\Theta_{-1}\|}\right)\|\Theta_{-1}\|^\alpha\right), \quad A\in\mathbb{B}^d \text{ with } \mathbf{0}\notin A,$$

and use an indirect estimator instead of a direct one.

Furthermore, we try to detect independence of $\|\Theta_1\|$ and $\Theta_1/\|\Theta_1\|$ and explore the implications of this fact for the structure of the spectral tail process.

References

Basrak, B. and Segers, J.: Regularly varying multivariate time series, *Stochastic Processes and their Applications* **119**, 1055–1080 (2009)

Drees, H., Segers, J. and Warchoł, M: Statistics for tail processes of Markov chains, *Extremes* **18**, 369–402 (2015)

Janßen, A. and Segers, J.: Markov tail chains, *Journal of Applied Probability* **51**, 1133–1153 (2014)

This is joint work with Holger Drees.