Spectral Theory and Applications

Symmetry of spectra of nuclear operators

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It was proved in a paper [1] by M. I. Zelikin: The spectrum of a nuclear operator A acting on a separable Hilbert space is central-symmetric iff $trace\, A^{2n-1} = 0$, $n \in \mathbf{N}$.

Recall that the spectrum of A is central-symmetric, if together with any eigenvalue $\lambda \neq 0$ it has the eigenvalue $-\lambda$ of the same multiplicity.

The part "only if" is evident if one applies the Lidskiĭ trace formula for trace class operators in Hilbert spaces [2]. An analogue of Lidskiĭ theorem for operators in Banach spaces is the famous Grothendieck trace formula for the 2/3-nuclear operators [3].

Our aim is to present the following generalization (Theorem 1) of both Grothendieck and Lidskiĭ theorems and *to apply this generalization* for getting an analogue of Zelikin's theorem for subspaces of quotients of L_p-spaces (Theorem 2). In a sense, these theorems can be considered as "interpolation results" in the scale "L_1 through L_2 to L_∞".

We need a definition: An operator T in a Banach space Y is s-nuclear, $0 < s \leq 1$, if T admits a representation

$$T = \sum_i \lambda_i y_i' \otimes y_i,$$

with $(\lambda_i) \in l_s$, (y_i') and (y_i) are bounded in Y^* and Y, respectively.

Theorem 1 [4]. Let Y be a subspace of a quotient (or a quotient of a subspace) of an L_p-space, $1 \leq p \leq \infty$. If $T \in N_s(Y, Y)$ (s-nuclear), where $1/s = 1 + |1/2 - 1/p|$, then

1. the (nuclear) trace of T is well defined,
2. $\sum_{n=1}^{\infty} |\lambda_n(T)| < \infty$, where $\{\lambda_n(T)\}$ is the system of all eigenvalues of the operator T (written in according to their algebraic multiplicities)

and

$$\text{trace } T = \sum_{n=1}^{\infty} \lambda_n(T).$$

Theorem 2. *Let Y be a subspace of a quotient (or a quotient of a subspace) of an L_p-space, $1 \leq p \leq \infty$, and $T \in N_s(Y, Y)$ (s-nuclear), where*

$1/s = 1 + |1/2 - 1/p|$. *The spectrum of T is central-symmetric iff* $\mathrm{trace}\, T^{2n-1} = 0, n = 1, 2, \ldots$.

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References

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