Spectral Theory and Applications

Two interacting particles on the half-line

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In this talk we discuss a system consisting of two particles moving on $\mathbb{R}_+$ and whose Hamiltonian is (formally) given by

$$H = -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + v(x,y)\left[\delta(x)+\delta(y)\right] \ ,$$

$v(x,y) = v(y,x)$ being some symmetric interaction potential. The considered two-particle interactions are of singular type, namely, the particles are interacting only whenever at least one particle is situated at the origin. Although this model originated from the theory of many-particle quantum chaos, it may also be of interest to other areas such as many-body quantum mechanics or applied superconductivity.

From a mathematical point of view, our system is translated into a boundary value problem for the two-dimensional Laplacian on $\mathbb{R}_+^2$ subject to Robin boundary conditions with variable coefficient. We will discuss spectral properties of H, i.e., we will describe the essential part of the spectrum and derive conditions on $v(x,y)$ which ensure the existence of at least one eigenstate below $\inf \sigma_{ess}(H)$. We will also estimate the number of such eigenvalues and provide an estimation of the ground state energy. Finally, we will prove exponential decay of the ground state in the case of $v(x,y)$ having compact support.

(This talk is based on joint work with T. Mühlenbruch (Hagen)).