Resources from "Unimodality, Log-concavity, and Beyond" March 16-20, 2020

We have posted some resources (slides, notes, videos) from the workshop. Titles and abstracts are given here, and the corresponding files are posted on the IML website. The hope is that collaboration can still occur between all of the participants, including those that could not attend due to COVID-19. If you have any questions or comments, or would like to contribute something to this page, please email Petter Brändén or Jonathan Leake.

Federico Ardila - Lagrangian geometry of matroids

Federico did not have slides prepared for the talk he was planning on giving, but instead sent slides from an earlier similar talk. The abstract for his originally planned talk is below.

We introduce the conormal fan of a matroid M, which is a Lagrangian analog of the Bergman fan. We also construct the bipermutahedron, a polytope whose normal fan contains the conormal fans of all matroids on a given ground set. We use these constructions to give a Lagrangian interpretation of the Chern-Schwartz-MacPherson classes of M. This allows us to express the h-vector of the broken circuit complex $BC(M)$ in terms of the intersection theory of the conormal fan of M.

We also develop general tools for tropical Hodge theory which allow us to prove that the conormal fan of a matroid satisfies the Kähler package: Poincaré duality, the hard Lefschetz theorem, and the Hodge-Riemann relations. We use this to prove the conjectures that the h-vector of $BC(M)$ is unimodal, log-concave, and flawless.

Peter Csikvari - Stability, Bethe approximation and some annoying problems

Stable polynomials and capacity bounds often provide lower bounds in combinatorial problems that are predicted by a statistical physics heuristics called Bethe approximation. In this talk I will explain these ideas and concepts, and show the limitations of this method. Finally, I will also give some open problems. This is a joint work with various subsets of Ádám Schweitzer, Márton Borbényi, András Imolay, Nicholas Ruozzi and Shahab Shams.

Chris Eur - Simplicial generation of Chow rings of matroids

Chris has an excellent video of himself giving this talk, which will be available until April 19th:

https://www.youtube.com/playlist?list=PLaMpLVOXsrG_FPvLJOKbFw8GMCQAzBkMO

Once unavilable, he has asked that anyone who wants to see it email him directly. The abstract is below.

In 2015 Adiprasito, Huh, and Katz settled Heron-Rota-Welsh conjecture that the coefficients of the characteristic polynomial of a matroid are log-concave. They proved this result by developing a combinatorial Hodge theory for the Chow ring of a matroid. We introduce a simple linear change of variables for the Chow ring of a matroid whose generators now yield a rich geometric and combinatorial interpretation via the theory of matroid quotients. As one application of this new presentation, we are able to obtain a new proof the log-concavity of the characteristic polynomial. This talk represents joint work with Spencer Backman and Connor Simpson.

Khazhgali Kozhasov - On complete monotonicity of inverse powers of some stable polynomials

Sufficiently high inverse powers p^a, $a < 0$, of some real stable polynomials p turn out to be completely monotone, that is, the coefficients of the Taylor expansion of $y \mapsto p(x-y)^a$ are nonnegative for any x in the positive orthant. I will discuss this phenomenon for elementary symmetric polynomials and products of linear forms. Based on a joint work with M. Mikhalek and B. Sturmfels.

Jonathan Leake - Approximate counting via polynomial capacity

The notion of the capacity of a polynomial was introduced by Gurvits around 2005, originally to give simplified proofs of the Van der Waerden lower bound for permanents of doubly stochastic matrices and Schrijver's inequality for perfect matchings of regular bipartite graphs. Since then, capacity has been utilized to bound various combinatorial quantities and to give polynomial-time algorithms to approximate such quantities. These types of results are often proven by giving bounds on how much a particular differential operator can change the capacity of a given polynomial. In this talk, we demonstrate this method via the example of bounding and approximating matchings in a bipartite graph.

Greta Panova - The Kronecker coefficients on many fronts

Greta has sent us notes for her talk, rather than the actual slides themselves.

The Kronecker coefficients of the symmetric group appear in different contexts throughout combinatorics, representation theory and even computational complexity theory. In this talk we discussed their connection with the number of integer partitions inside a rectangle where certain Kronecker coefficients are expressed as their consecutive differences (and so reprove Sylvester's unimodality theorem). We use probabilistic methods to derive thigh asymptotics for these differences and hence the Kronecker coefficients themselves. We then discuss their role in Geometric Complexity Theory, where the distinction of computational complexity classes ultimately boils down to compression between representation theoretic constants like the Kronecker coefficients.

Mohan Ravichandran - Generalized permutahedra: Minkowski linear functionals and Ehrhart positivity

Generalized permutahedra form a combinatorially rich class of polytopes that naturally appear in many areas of mathematics such as combinatorics, geometry, optimization and statistics. They also comprise many important classes of polytopes, for example, matroid polytopes. We study functions on generalized permutahedra that behave linearly with respect to dilation and taking Minkowski sums. We give a complete classification of all positive, translation-invariant Minkowski linear functionals on permutahedra that are invariant under permutations of the coordinates: they form a simplicial cone and we explicitly describe the generators. We apply our results to prove that the linear coefficients of Ehrhart polynomials of generalized permutahedra are nonnegative, verifying conjectures of De Loera-Haws-Koeppe (2009) and Castillo-Liu (2018) in this case. This is joint work with Katharina Jochemko.